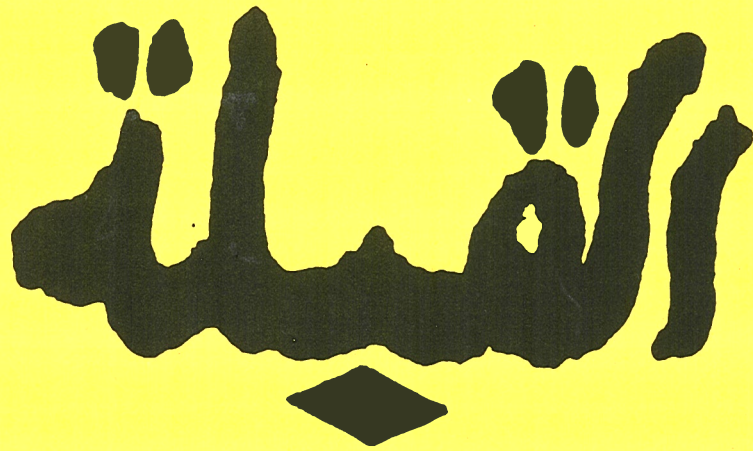


MECCA



KIBLA
Arabic for "to Mecca"

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Worksheets

1. Arrows to Mecca (chapter 1)
2. Airlines to draw (chapter 4)
3. Kibla map (chapter 4)
4. Compare solid geometry and spherical geometry

Press cuttings and other information about Mecca, to use in a project.

1. Boeing bound for Seoul (NRC, 16 Oct '90)
2. New mosque misses Mecca by three degrees (EC, 1 Nov '90)
3. Europe to Seoul: seven and a half hours faster (advertisement, 5 May '90)
4. Kibla (Encyclopaedia of Islam, Volume V, p. 84) (plus a description)
5. A notice, a cartoon and the Surinam Brethren (radio, Jan '91)
6. The sun as a balloon over Mecca
7. 'Composed track': an alternating use of central projection and mercator projection in navigation

CHAPTER 1: Where is Mecca?

1. The Imam

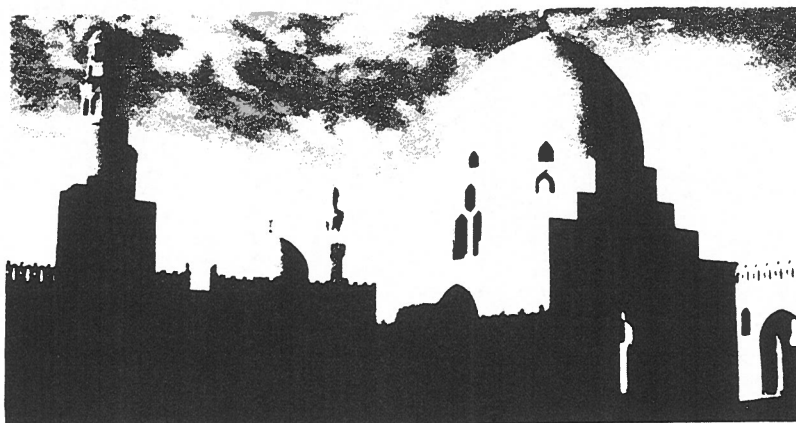
In the mosque the faithful say their prayers in a certain direction.

Do you know in which direction?

How do they know which direction is the right one?

I did not know.

I asked the Imam of a mosque.



a mosque

When the Imam appeared I asked him straight away:

"When you pray, you pray towards the East, don't you?"

"No we don't, we pray towards Mecca" was his answer.

"But how do you know where Mecca is? I can find the East, but Mecca ...?"

"You can find Mecca in three different ways". And he explained:

"First of all you can look at the sun. But that may be a problem. Very often you can't see the sun at all here and in summer the sun rises in a different place than in winter.

Better is therefore to use a kind of compass which points towards Mecca.

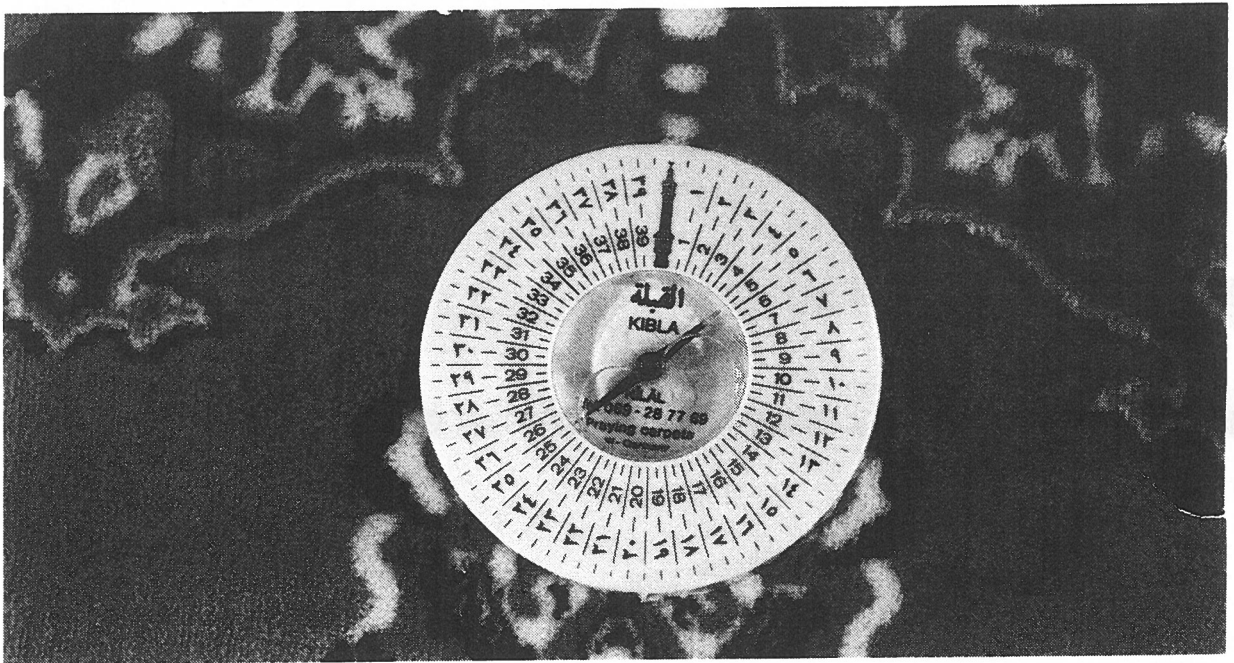
And thirdly: if you can't see the sun and you don't have the compass, you can still have a feeling for the right direction of Mecca".

"Does a Moslem in Kaapstad pray in a different direction than for instance a Moslem in Amsterdam or a Moslem in Havana?"

The Imam nodded: "Yes. From every place on earth one must pray in another direction to Mecca".

1. Do worksheet nr 1. Arrows to Mecca

Suddenly the Imam asked me to wait for a moment. He disappeared, and came back with a beautiful little carpet. A kind of compass was attached to the middle of the carpet, a *Mecca-meter*.



the carpet with the compass

In the Mecca-meter was written: "Kibla"; it means: "to Mecca".

"That needle is always pointing north" I said, "not to Mecca; how do we know where Mecca is"?
Underneath the carpet was a little bag and from the bag the Imam took a booklet.

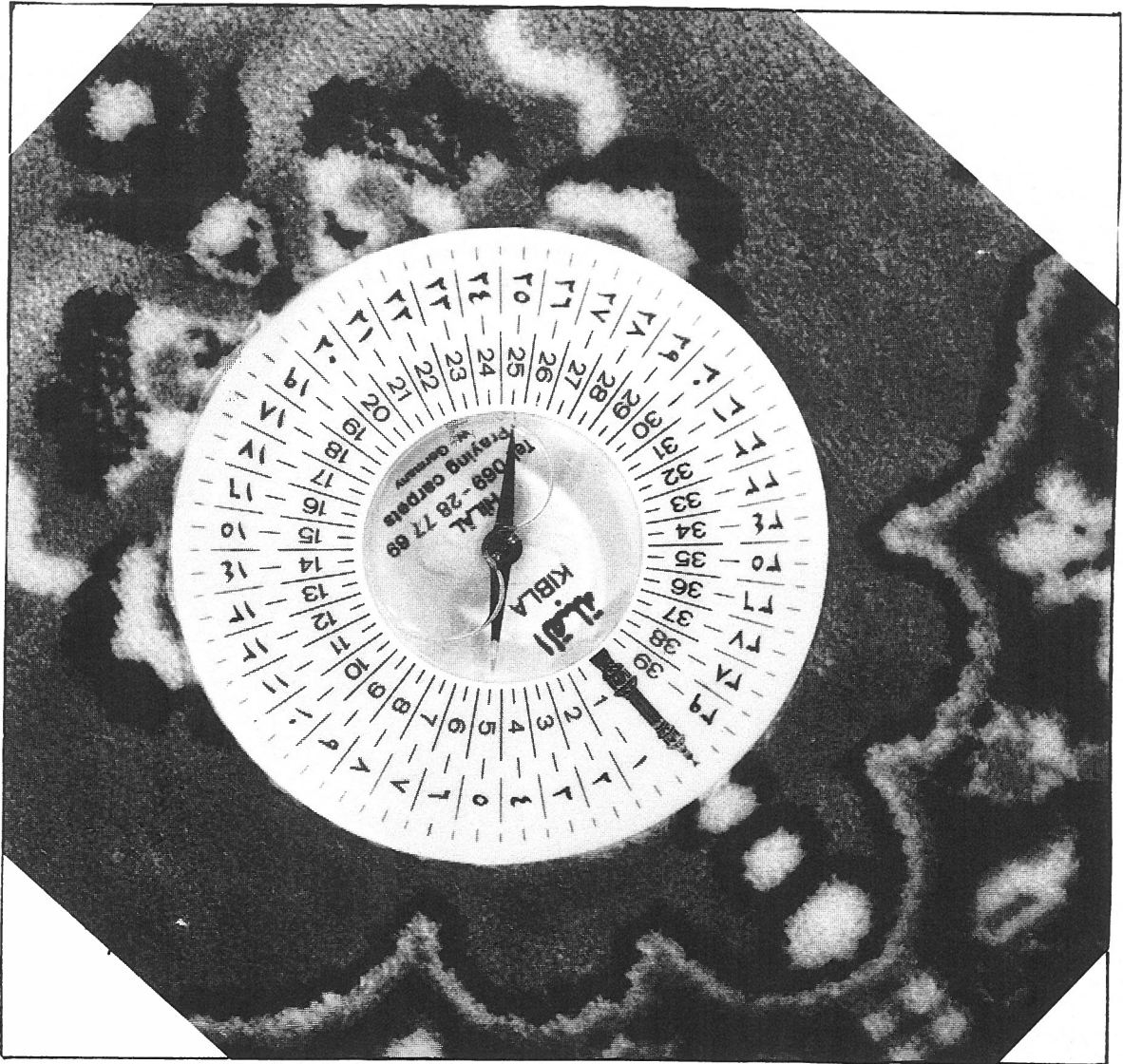
In the book cities plus numbers were printed, like this:

250 Amsterdam

300 Havanna

100 Hong Kong

The Imam read out to me "250 Amsterdam" and turned the carpet until the needle of the compass pointed to the number 25. That is 250. Like that".



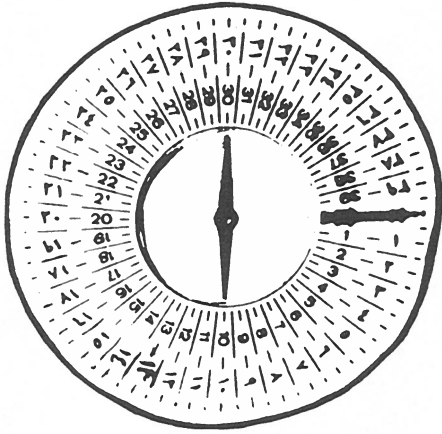
the carpet with the needle on 25

And then the Imam could show me in which direction Mecca was:

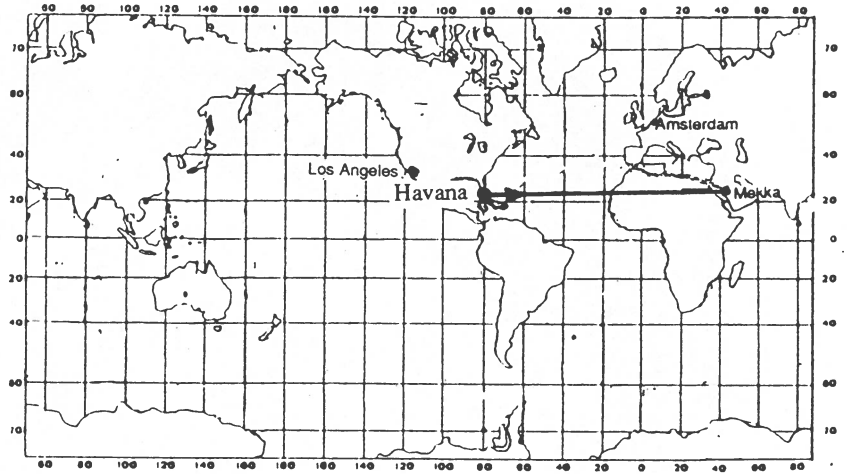
"Kibla!

The minaret (church tower) on the Mecca-meter is pointing towards Mecca".

2. In the booklet is written: 300 Havana
 If you are in Havana, then the needle should point to 30.
 If you are in Havana, are you looking eastwards or westwards?
 Write it down in your note-book.



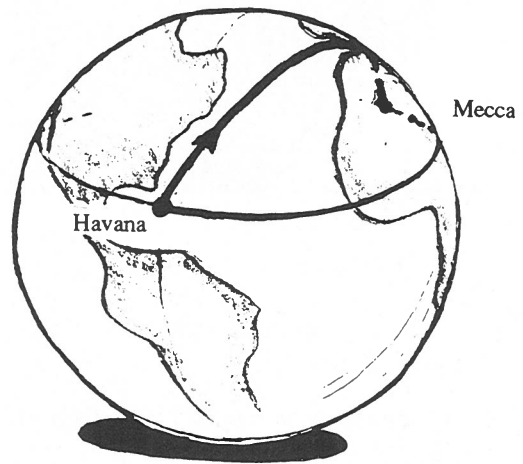
Mecca-meter.



worldmap with Mecca, Havana and Amsterdam

2. A directional problem

Other Arab scholars think that from Havana one does not look eastwards, but northeast.



earth with Mecca and Havana

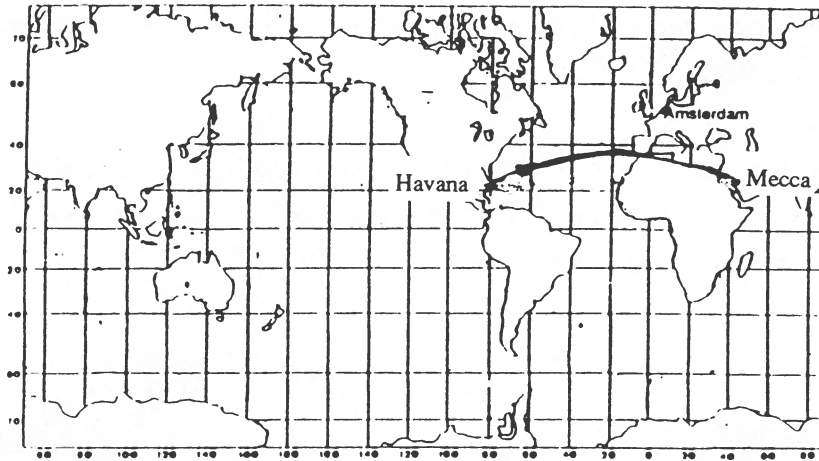
That is because they do not think of a map of the world, but of the globe.

3. What do you think?

From Havana the direction of Mecca is east, or northeast?

Or could it be any direction?

4. An airplane from Havana to Mecca does not fly eastwards straight away, but goes northeast first. Why?



worldmap with Mecca and Havana

3. Summary

This book is about the differences between the map of the world (which is flat) and the globe (which is spherical).

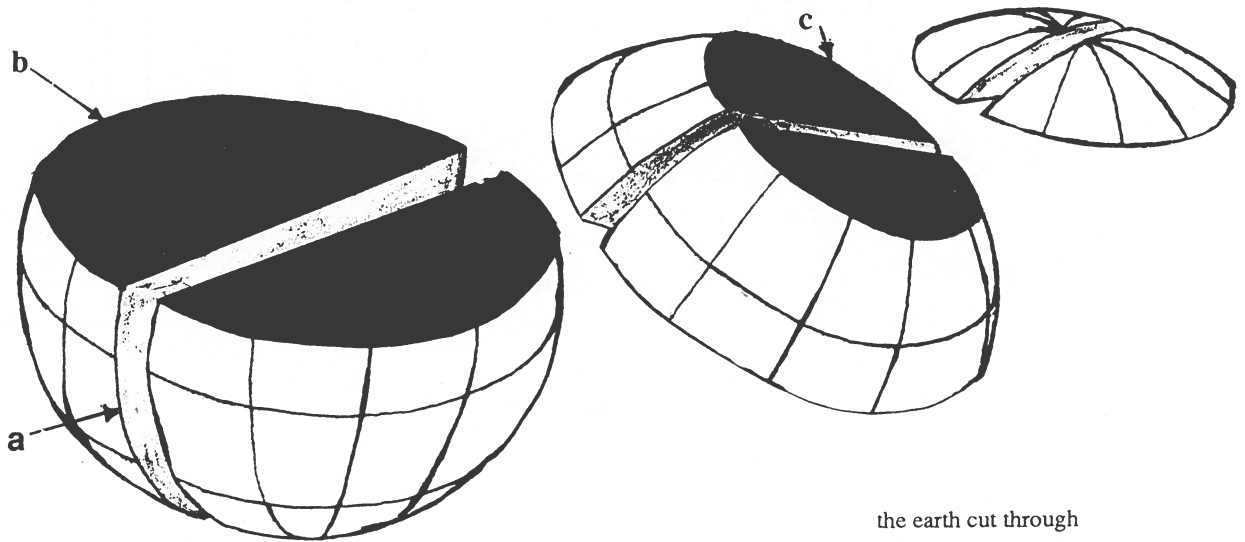
It is about differences in direction and distance.

CHAPTER 2: Great circles

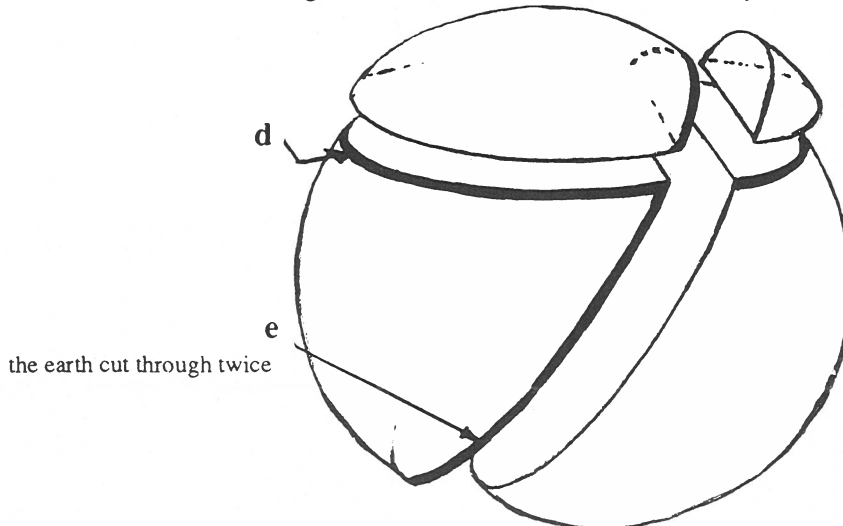
1. Secant circles and great circles

We are going to make as if we cut the earth right through, just make believe. While you are cutting your knife may not wobble, it must be an absolutely straight cut. If you cut the earth in two, the cross-section will be a circle.

5. If you cut through the earth at two different places, will the secant circles be equal in size? What do you think?
6. You know that the equator, the parallels and the meridians are circles on the earth. Look at the figure: along which kind of circles was the earth cut in two? Write the answers in your note-book.



7. Which circle is the greatest: secant d or secant e? Why?



8. Secant circles are not always of the same length.
How long is the longest secant on the globe?
9. Are there secant circles on the globe that will be longer than the equator?
Yes/No, because(note-book)
10. The equator is a "great circle". Can you think of other great circles on the globe?
Give an example, or otherwise write why other great circles are not on the globe.
11. How would you explain what a great circle on the globe is?
12. Here are some answers of other pupils. Which answers are right? And which are wrong?
- a. *a great circle is a circle that divides the globe in two halves.*
right/wrong, because
- b. *a great circle is the greatest possible circle on the globe.*
right/wrong, because
- c. *a great circle is a circle that divides the globe in two pieces.*
right/wrong, because
- d. *a great circle is a circle that goes through the north and south pole.*
right/wrong, because
- e. *a great circle is a circle whose centre is the earth's centre.*
right/wrong, because
- f. *if you keep going straight while going around the earth, you'll make a great circle.*
right/wrong, because
- g. *a great circle is a circle around the earth that is 40000 km long.*
right/wrong, because
- h. *a great circle is a circle on the earth that is not parallel to the equator.*
right/wrong, because

Remember:

a great circle is a circle whose centre is the centre of the earth.

13. Is a parallel a great circle?

14. Where are the parallels' centres situated?

2. Great circles and points on the globe (pole and anti-pole).

Investigate with the help of balls, for example polystyrene balls or volley balls.

15. How many great circles can you draw through the north pole?

16. How many of the great circles that go through the north pole, will also go through the south pole?

17. How many of the great circles that go through the north pole, will pass through Amsterdam?

18. We could choose another point on the globe as a "pole".

For instance Amsterdam (52° north latitude, 5° east longitude).

- a How many great circles will pass through Amsterdam? (note-book)
- b Will Amsterdam have an "antipole", just like the north pole has the south pole?
- c Where is that "antipole" of Amsterdam?
- d Will every great circle through Amsterdam also pass through the antipole of Amsterdam?
- e Is there a great circle through Amsterdam that will also pass through the south pole?
- f Will that same great circle then go through the north pole?

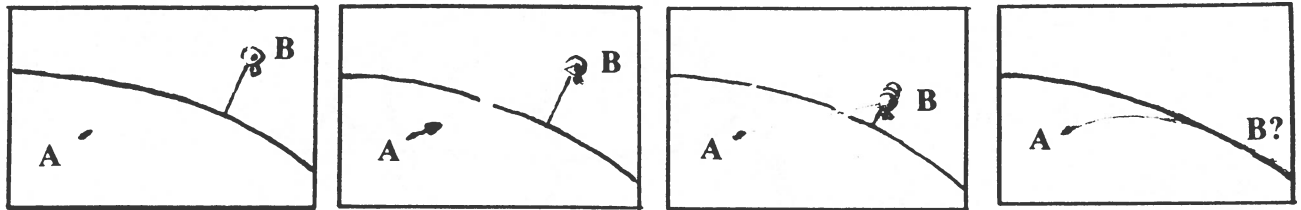
19. We can also choose Mecca (21° north latitude, 40° east longitude) as a "pole".

- a How many great circles will pass through Mecca?
- b Will Mecca also have an "antipole"?
- c Where is that "antipole" of Mecca?
- d Will every great circle through Mecca also pass through the antipole of Mecca?
- e Is there a great circle through Mecca that will pass through the south pole?
- f Will that same great circle then also pass through the north pole?

- g Is there a great circle through Mecca that will pass through Amsterdam?
- h Will that same great circle then also pass through the antipole of Amsterdam?

CHAPTER 3: Great circle, direction and shortest distance

1. A balloon over Mecca : direction Mecca



- | | | | |
|-------------------------------------|---------------------------|--|---|
| <p>1.
A balloon!
Where?</p> | <p>2.
Over there!</p> | <p>3.
They're pulling
it down.</p> | <p>4.
It's disappeared!
But I know the
direction now.</p> |
|-------------------------------------|---------------------------|--|---|

Suppose you're pointing to balloon **B** from position **A**.

Then you will be pointing along a straight **vision line** (like a laser beam, for instance) to balloon **B**.

If balloon **B** is pulled down, your vision line will describe a vertical vision plane.

That vision plane will intersect with the globe along a **secant plane**.

And this secant plane includes your eye and the descending balloon.

31. A point in the city of Mecca will also be included in this secant plane.

Yes/No, because

32. Will the centre of the earth also be included in this secant plane?

Yes/No, because

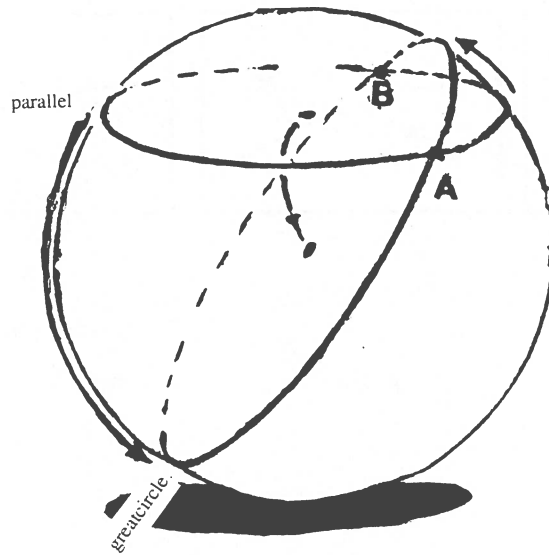
The straight vision line **AB** cuts through the earth like a knife.

In figure 4 balloon **B** is touching the ground.

On the earth you will observe a curved intersecting line which connects **A** with balloon **B** on the ground.

33. Why is that curved intersecting line **AB** part of the *great circle* through **A** and **B**?

26. Suppose: **A** and **B** are points on the northern hemisphere of the globe.
 The short arc **AB** on the *great circle* will always be on top of the short arc **AB** on the *parallel*, if **A** and **B** are in the northern hemisphere.
 Yes/No, because (look at the position of the centres of the circles)



27. Can you think of another circle through **A** and **B**, bigger than the great circle?
 Yes/No, because (look at the position of the centres of the circles)
28. Will the antipole of point **A** be on the great circle through **A** and **B**?
 Yes/No, because
29. Suppose: **A** and **B** are points on the **southern** hemisphere of the globe. What is true:
 the short arc **AB** on the *great circle* will always be
 on top of / under
 the short arc **AB** on the *parallel*.

The direction from A to B on the globe.

30. Which circle (parallel, small circle, great circle) will give you the direction in which to go from **A** to **B** on the globe?
 Or can you go from **A** to **B** via any circle?

4. Summary

Through two points on the globe you can imagine a revolving door, a cluster of circles of which one is the greatest circle.

A *great circle* is a circle with its centre in the centre of the earth.

The *antipole* of a point on the globe - where is it?

20. Devise a rule concerning great circles and points on the globe.

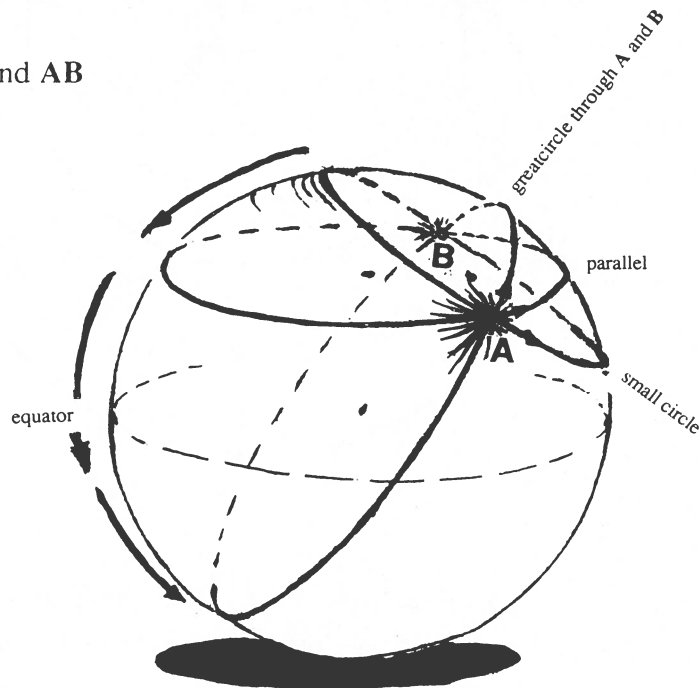
(Hint: when will you have lots and lots of great circles through two points? When will you have exactly one great circle through two points? Etc.)

3. A cluster of circles through two points on the globe.

A and **B** are on one parallel. Think of Havana and Mecca for example.

We can draw some other secant circles through **A** and **B**: a great circle, a small circle and anything in between. A *revolving door* around **AB** will cut the globe in secant circles.

Revolving door around **AB**



The secant circles will get bigger and bigger, and then again smaller and smaller as they cut through the wall of the globe.

21. On which circle (parallel, small circle, great circle) is the arc **AB** going to be the smallest?

22. On which circle will arc **AB** be the biggest?

Centres

Pay attention to the centres of the circles through **A** and **B**.

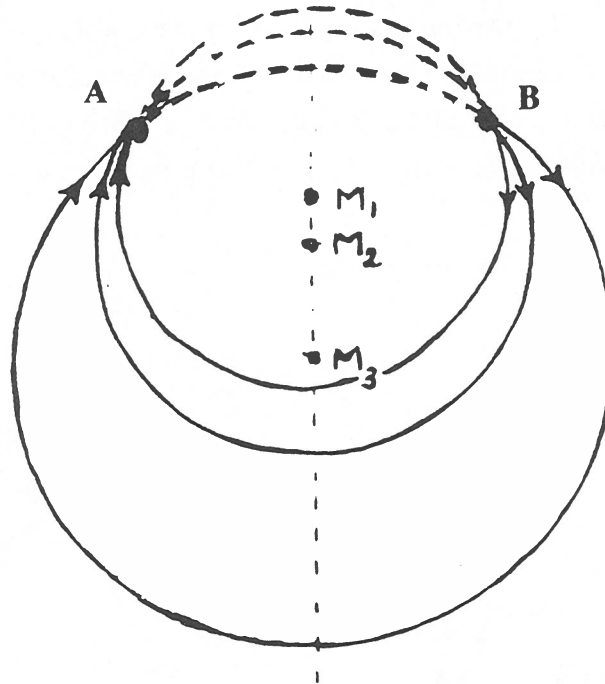
23. Where is the centre of the small circle through **A** and **B**?

24. Where is the centre of the great circle?

25. Where is the centre of the parallel?

2. Drawing circles through two points: the great circle and the shortest distance

Circles through two points **A** and **B**:



34. Take your pair of compasses and your ruler. Copy this drawing in your note-book.
Remember: every circle should pass through **A** and through **B**.

35. How will you call the line on which the centres M_1 , M_2 , M_3 are ?

36. Which of the three circles (M_1 , M_2 , M_3) will give you the shortest arc between **A** and **B**?

37. Which of the three circles (M_1 , M_2 , M_3) will give you the longest arc between **A** and **B**?

38. Can you think of a circle on earth that will produce the longest arc possible between **A** and **B**?
Yes/No, because (note-book).

CHAPTER 4: Great circles on a map

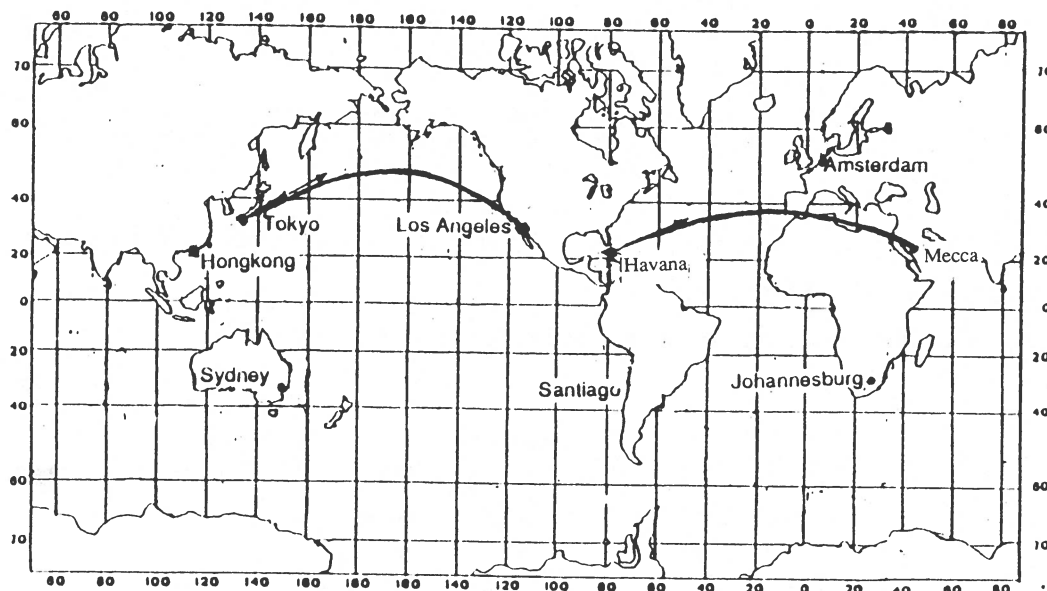
On a globe the great circle between two points will indicate the shortest distance between these points plus the direction to take from one point to the other for this distance.

But drawn on a map (a map being flat) the great circle does not seem to do this.

1. Airlines in elastic (shortest distance)

An airplane will fly along a great circle because that is the shortest distance.

You see here a map of the world where upon the route Tokyo - Los Angeles and the route Havana-Mecca are indicated.



44. Are these indeed the shortest distances on earth between Tokyo and Los Angeles and between Havana and Mecca?

Check this on a globe with a piece of paper.

The shortest distances on the globe will often show as curved lines on a map.

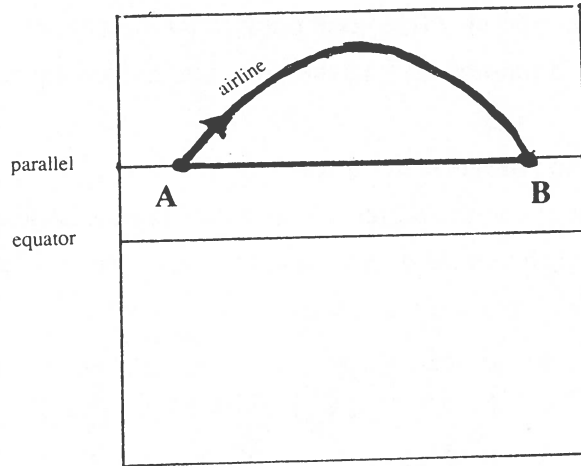
They will seem longer than the straight lines on the map.

45. Take an airplane, going from Amsterdam to Los Angeles. In which direction will it take off? More or less northward? Can you believe it?

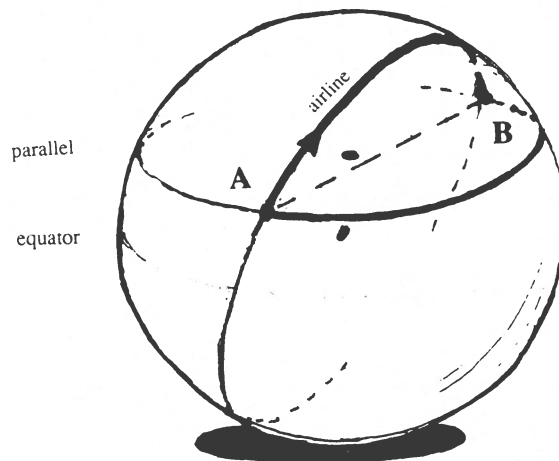
Try to decide that question with the help of a piece of paper on a globe.

2. Globe and map of the world.

A and B are points situated in the northern hemisphere. They are also situated on the same parallel.



46. In the northern hemisphere the airline between A and B will be north of the parallel between A and B. Why is that?
(Remember the "revolving door" of circles between A and B)

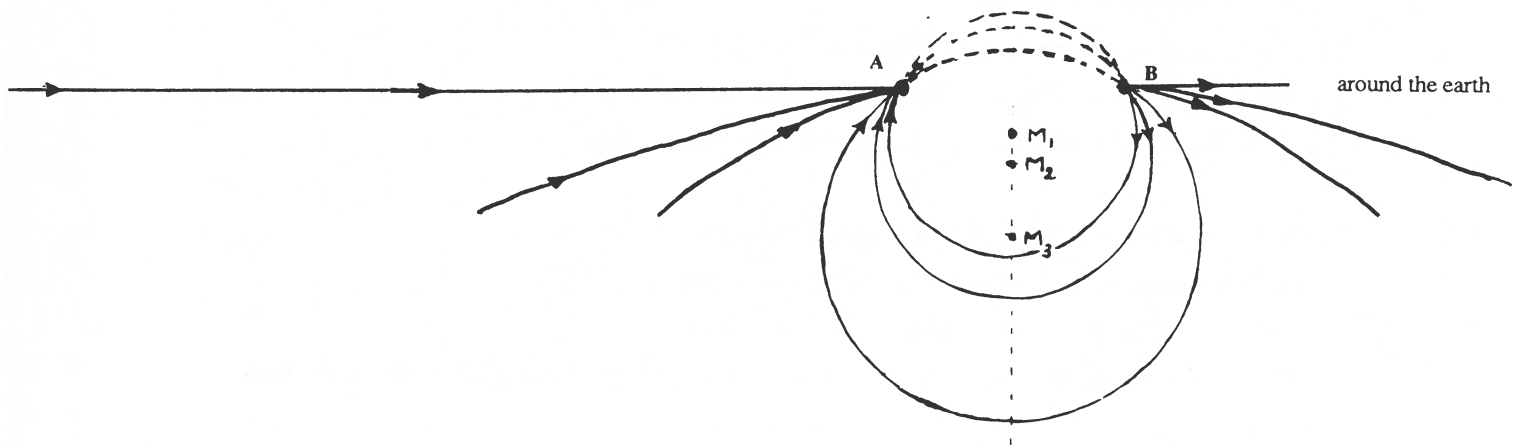


47. Do worksheet nr 2: Airlines.
Use a globe and a piece of paper.

48. What about the airlines in the southern hemisphere?

Theo's story

Somebody in my class said that if you started here on your paper, at this point **B** and you moved to the right all the time, off the paper, *underneath the earth*, and then again coming up on the left side, to **A**; that would be the greatest circle you could think of on earth, going through the two points **A** and **B** on your paper.

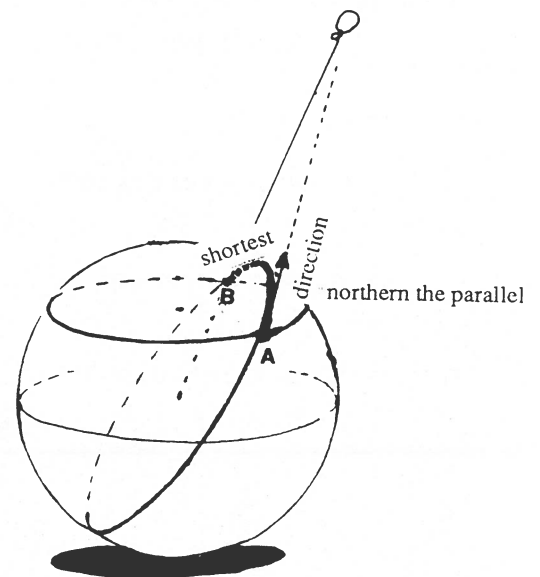


39. What is your opinion of Theo's story?
40. Draw in your note-book the **shortest** distance from **A** to **B**.
41. Will that shortest distance also be on the great circle through **A** and **B**?
Yes/No, because
42. If you know which is the shortest distance between two points, will you then at the same time also know the longest distance along a straight line between these two points on earth?
In wich direction have you to go?
43. Formulate two rules about **the shortest distance** and **the direction** between two points on the earth.
Remember the great circle through these points.

3. Summary

The *shortest distance* between two points on the globe is along the great circle through these points.

The *direction* between two points on the globe is dictated by the *great circle*.



Student Text

49. Which direction will an airplane take to go from one point on the equator to another on the equator?

North of the equator/South of the equator/Along the equator, because

50. A pilot will take the shortest distance from **A** to **B**.

On the world map that is a straight line. Where may **A** and **B** situated on the earth?

3. Kibla map of the world

The Kibla map is a world map filled with little arrows.

The arrows depict the direction to Mecca *via great circles*.

Do the following exercises. Use worksheet nr 3 Kibla map.

51. Draw on the Kibla map the great circle from Havana to Mecca.

Pay attention to the *rows* of arrows you encounter.

First a *vertical* row, then again a *horizontal* row.

The arrows indicate how to draw further. You have to adjust a little bit all the time.

52. To get to Mecca from Havana, what would your initial direction be?

(measure the angle between the direction north and the arrow in Havana)

53. And, having started, will you have to adjust your direction,

or can you keep going in the same direction to get to Mecca via the great circle?

54. Colour in the antipole of Mecca on your Kibla map. Why is this your antipole?

55. "I am going straight from **A** to **B**" said the pilot. He chose the correct starting direction to steer and kept going in that same direction. Will he arrive in **B**, do you think?

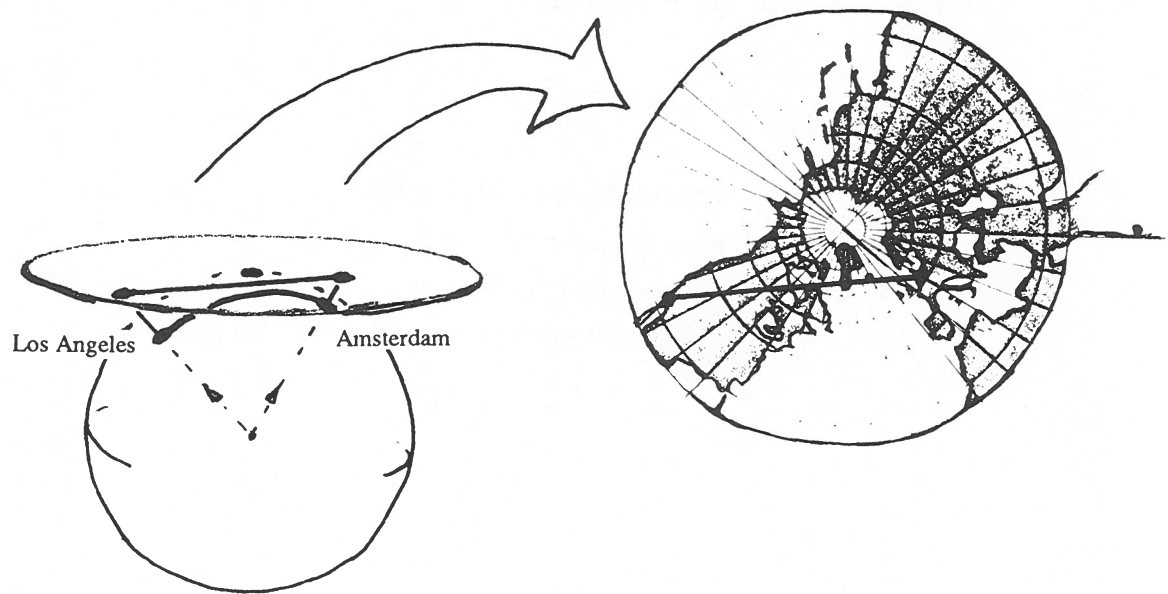
4. Polar map of the world

56. A airplane wants to fly along a great circle. Will it have to adjust its course all the time?

Maps of the earth exist where upon *all* great circles are straight lines.

57. If an airplane uses those maps, will it still have to adjust?

Here you can see a map like that: a polar map of the North Pole.
 The points on the earth have projected from the earth's centre to the map.



central projection

On this map the route Amsterdam/Los Angeles will be a straight line (Why?).

58. How do we call the straight lines through the North Pole?
59. Demonstrate that the pilot using this map also has to adjust?
60. Why is the great circle a straight line on this polar map?
 (Remember: a great circle is a secant circle with its centre in the earth's centre)

5. Summary

The shortest distance (great circle) between two points on the globe is usually a *curved* line on a world map. In the northern hemisphere this line will curve slightly north and in the southern hemisphere it will curve slightly south.

On a polar map (central projected) the greatcircle is a *straight* line.

If you fly or sail along a great circle you will have to *adjust* your course continuously (with respect to north).

6. Solid geometry and spherical geometry

These two types of geometry are different. A straight line in solid geometry is comparable with a great circle on the sphere. There are more differences.

61. Do worksheet 4: Compare solid geometry and spherical geometry

CHAPTER 5: Worksheets

Worksheet 1 : **Arrows to Mecca** (chapter 1)

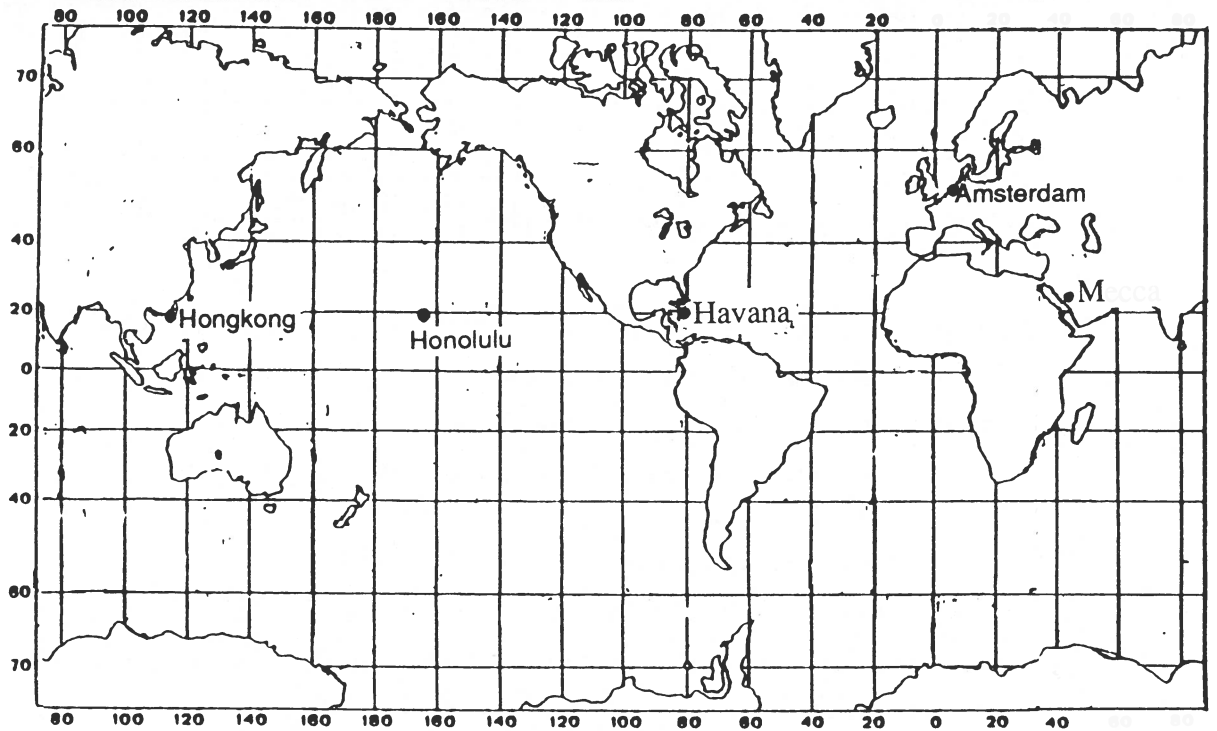
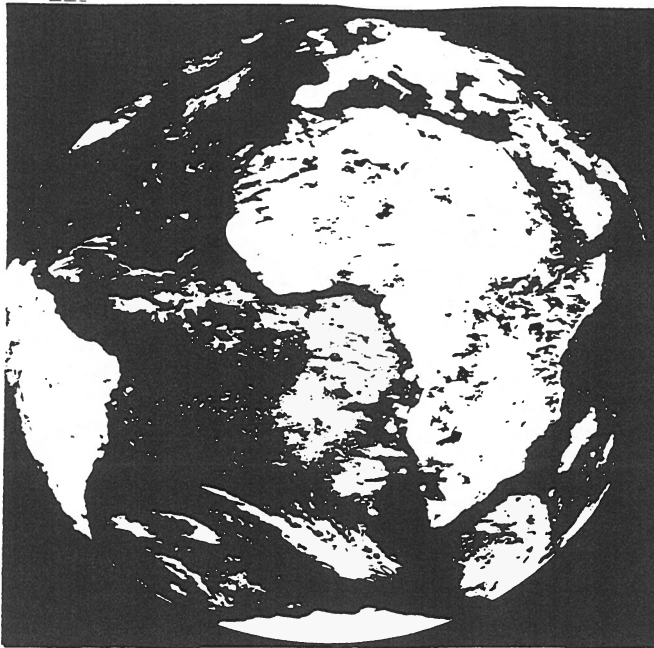
Worksheet 2 : **Airlines** (chapter 4)

Worksheet 3 : **Kibla map** (chapter 4)

Worksheet 4: **Compare solid geometry and spherical geometry**

Worksheet 1 : Arrows to Mecca.

Chapter 1 - exercise 1



1a. Draw the place of Mecca and of Amsterdam both on the satellite picture and on the globe.

1b. Colour in the equator on the map, on the picture and on the globe, in red.

1c. Draw arrows to Mecca on the map :

Amsterdam/Mecca,

Havana/Mecca,

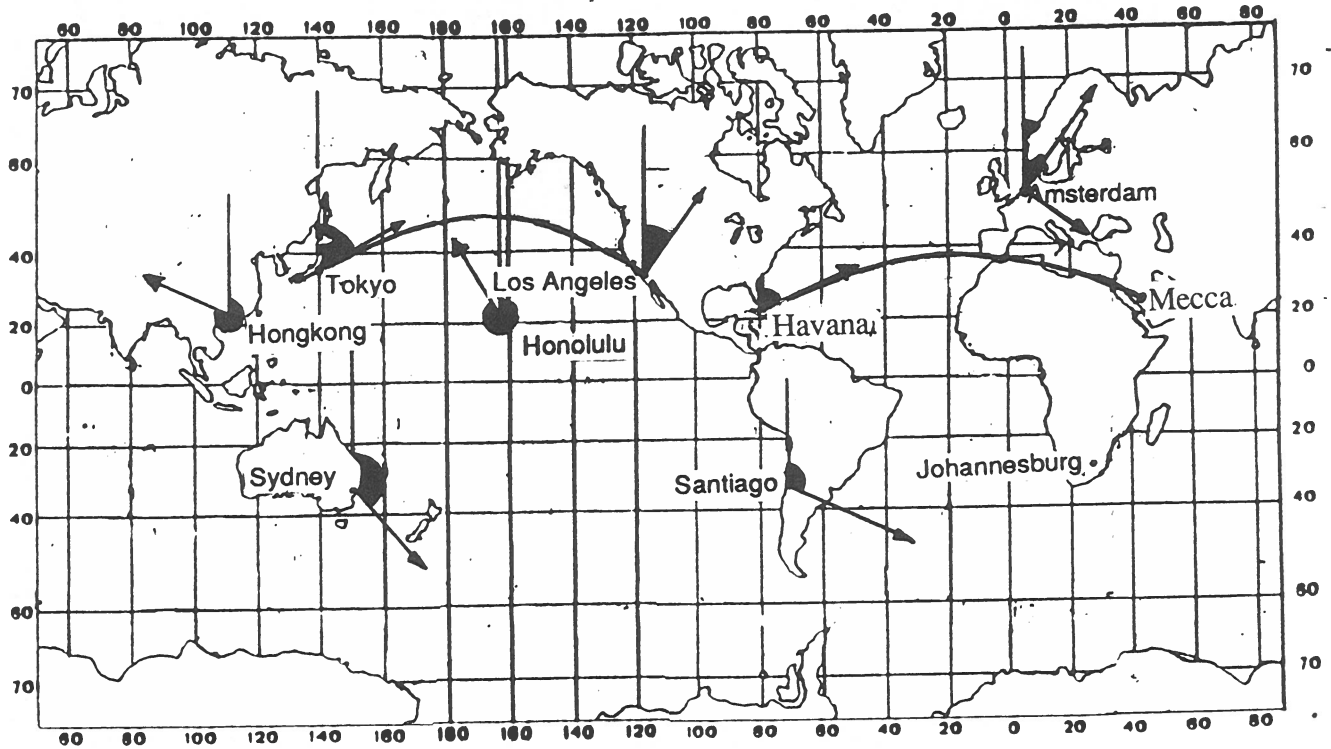
Hong-Kong/Mecca (remember - the earth is round)

Honolulu/Mecca.

Worksheet 2: Airlines

Chapter 4 - exercise 47

Drawing exercise



In this map two airlines have been drawn: Tokyo/Los Angeles and Havana/Mecca. They are the shortest distances on the globe (great circles), but they are not straight lines on this world map.

47a. Draw the following airlines on the world map:

Los Angeles/Amsterdam (the starting direction is indicated by an arrow at LA)

Sydney/Santiago

Santiago/Johannesburg

Draw these airlines on the world map. To be able to find the line the airplane will follow, use a slip of paper on the globe. Then draw that line on your map. The starting direction is already indicated by a small arrow.

47b. Do the same with the following airlines:

Hong Kong/Mecca (which will go over the edge of the map)

Honolulu/Mecca

Amsterdam/Tokyo

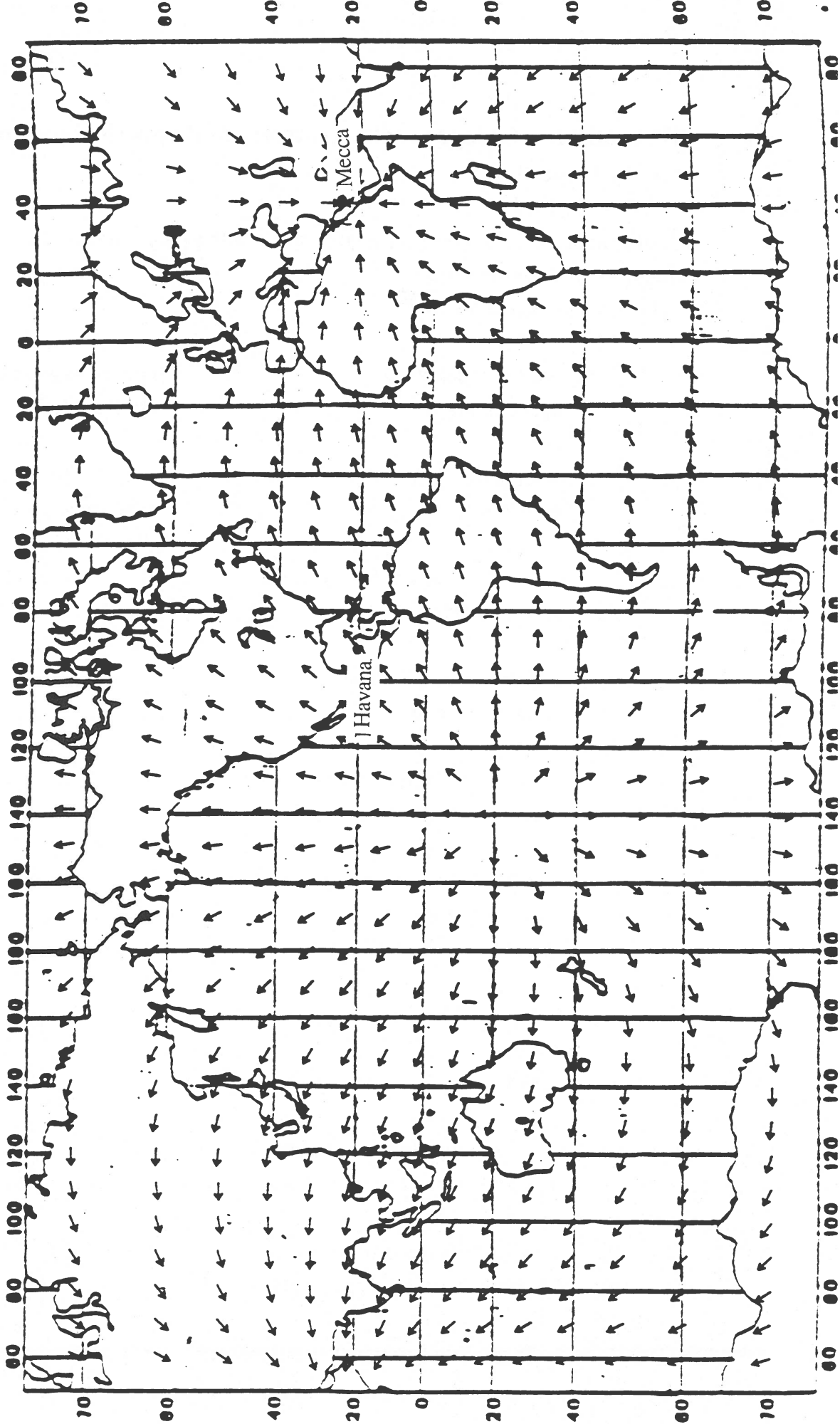
47c. Think of and draw an airline yourself from a place in the northern hemisphere to one in the southern hemisphere (for example from Los Angeles to Sydney).

47d. Draw an airline in the north-south direction (for instance from Amsterdam to Johannesburg).

Will that also be a curved line?

Worksheet 3: Kibla map

Chapter 4 - exercise 51



Worksheet 4: Compare solid geometry and spherical geometry

Chapter 4 - exercise 61

A straight line in solid geometry is comparable with a great circle on the sphere.
There are more differences.

Complete the second column

solid geometry in the plane

point in the plane
straight line

1. Infinite number of straight lines
through a point

2. Only one straight line
through two points

3. Two straight lines have one point
in common or are parallel

4. A point P outside straight line L.
Just one straight line
goes through P parallel to L

5. A point P outside straight line L.
Just one straight line
goes through P at right angle to L

6. Two points divide a straight line
into three parts

7. Only one of these three parts can be used
for defining distance of two points

8. There is no greatest distance between
two points

spherical geometry on the sphere

point on the sphere
great circle

1.
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8.
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CHAPTER 6: Press cuttings and other information about Mecca - to use in a project.

Assignment

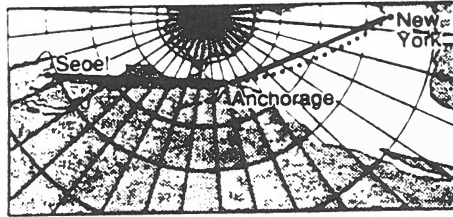
Do a mathematical project; work on your own or work together in a small group.

Make a selection from the following articles.

Use the mathematics we learned in this last section.

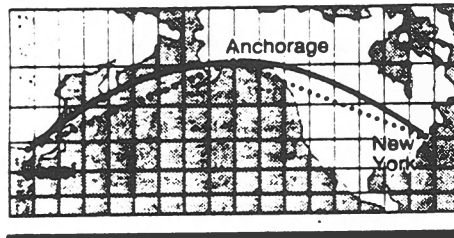
1. Boeing bound for Seoul (NRC. 16 October 1990)
2. New mosque misses Mecca by three degrees (Emmer Courant, 1 November 1990)
3. Europe to Seoul: seven and a half hours faster (advertisement, 5 May 1990)
4. Kibla (Encyclopaedia of Islam, Volume V, p 84) (plus description)
5. A notice, a cartoon and the Surinam Brethren (radio, January 1991)
6. The sun as a balloon over Mecca
7. 'Composed track': an alternating use of central projection and mercator projection in navigation

1. Boeing bound for Seoul (NRC, 16 October 1990)



polar map

In 1983 a Boeing of Korean Airlines was shot down over Sachalin. In most newspapers the route of this Korean Boeing was depicted on a world map (see bottom picture). Lots of people will have wondered why an airplane on its way from New York to Seoul is flying over Alaska. When we look at the uppermost picture (the polar map) we will understand why.



world map

ALL world maps tell lies.

**2. New mosque misses Mecca by three degrees
(Emmer Courant November 1990).**

Emmen - Because of incorrect measuring in preparation for the building of a mosque at the Peijserhof in the Emmen district of Angelslo the municipality could be forced to issue a building permit for the third time in a row. The Turco - Islamic cultural society has discovered that the foundations for the building have been laid out incorrectly with regard to the holy city of Mecca. Considerations concerning changes in the building plans are in progress.

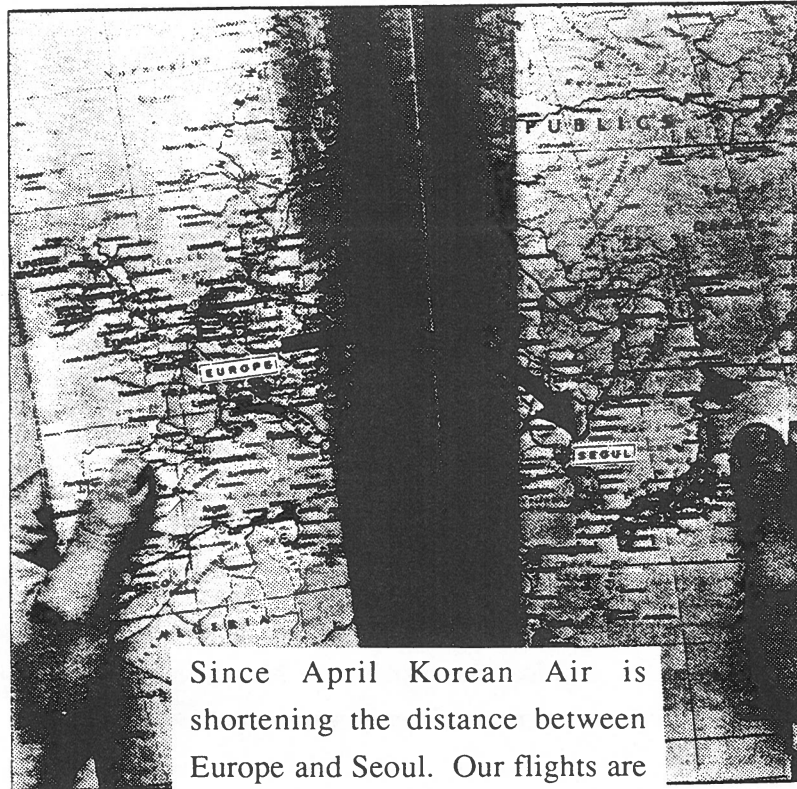
According to a committee of concerned neighbouring citizens (protesting the building of a mosque) a new building permit will have to be granted in case of any changes in the building of the mosque.

According to A.S. Heine, employee at the department of Town Planning, a "marginal" shift is all that is necessary. "As far as I can see now we just have to shift the foundations of the original plan by two or three degrees. In that case building can proceed according to the present permit, unless severe technical adaptations are in order, or building the mosque will overstep the limits of the town-plan."

The municipality will not hesitate to issue a new building permit, if necessary.

3. Europe - Seoul. Seven and a half hours faster (advertisement May 5th 1990)

EUROPE - SEOUL
7 1/2 HOURS FASTER



Since April Korean Air is shortening the distance between Europe and Seoul. Our flights are getting shorter by seven and a half hours. Simply by flying a different route. In this way you gain a whole working day.

AN EXTRA DAY

TO DO YOUR BUSINESS IN.

You are already landing at Seoul while others are still only passing Anchorage. In this way you'll also get quicker to Japan and to South-East Asia. And added to this, don't forget the pleasant comfort of our Prestige Class. Korean Air : the best and fastest way to get to Seoul.

4. Kibla (Encyclopaedia of Islam, Volume V, p.84)

In the old times Arabic scientists used different methods to find the Kibla.

Here is a description of a way to find the Kibla (more or less), starting in Amsterdam for example.

astronomers. Indeed, the *kibla* problem may be transferred to the celestial sphere simply by considering the zenith of Mecca rather than Mecca.

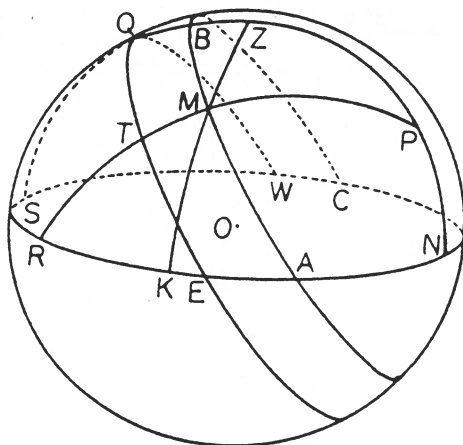


Fig. 2

zenith of Mecca M on the celestial sphere O. The local horizon is NESW. The celestial equator is NPZS. EQW is the celestial meridian of Mecca. Now:

$$\sin \phi_M, \text{ and } QT = \Delta L,$$

to determine the azimuth of Mecca from the local horizon by the arc SK = q. To solve this problem first find the distance of Mecca above the local horizon by the arc MK = h. This involves the problem of determining the distance between two points on a sphere. This is mathematically equivalent to the problem of finding the distance between two points (φ, δ, t) of a sphere. The distance is measured by the arc QT (measured by PN). It is simply that the local declination angle is ΔL. The locality is

ments contain a chapter on the determination of the *kibla*.

Approximate solutions

A popular approximate method for determining the *kibla* which occurs in the *Zīdj* of the Syrian astronomer al-Battānī (fl. Raqqā, ca. 297/910) and in several unsophisticated Islamic astronomical works such as *al-Mulakkhḥaṣ fi 'l-hay'a* by al-Djaghmini (fl. Khwārazm, ? ca. 725/1325) is the following.

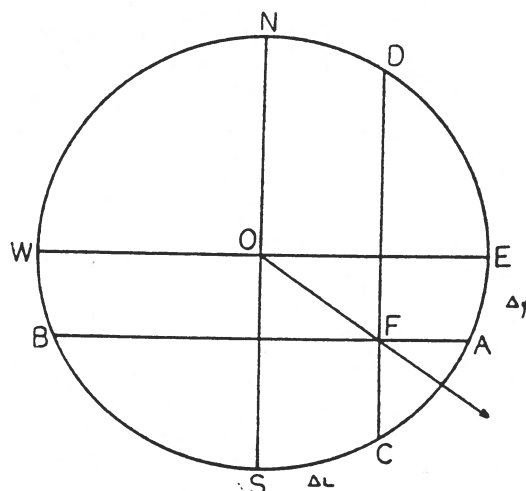


Fig. 3

Fig. 3 shows the construction for a locality where Mecca is to the south east. Mark the cardinal directions NWSE on a horizontal circle centre O and radius R and measure arcs EA=WB=Δφ=φ-φ_M southwards and SC=ND=ΔL eastwards. Next draw AB and CD, and denote their point of intersection by F. Then OF defines the *kibla*. This method is equivalent to an application of the formula

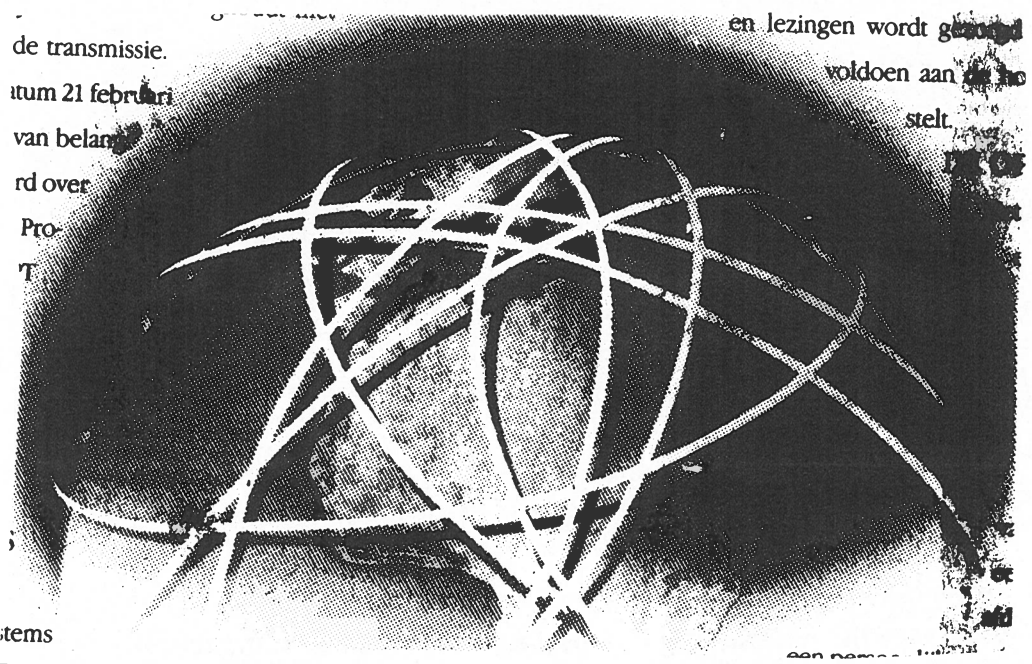
$$q = \sin^{-1} \left\{ \frac{R \sin \Delta L}{\sqrt{\sin^2 \Delta \phi + \sin^2 \Delta L}} \right\} = \tan^{-1} \left\{ \frac{R \sin \Delta L}{\sin \Delta \phi} \right\}$$

1. Amsterdam will be point O on your paper.
2. Draw a line north/south through O and a line east/west.
3. Draw a circle with centre O. This is the horizon around Amsterdam.
4. Compare the coördinates of Amsterdam (52° N 5° E) to those of Mecca (21° N 40° E) : Mecca is 35° more east than Amsterdam, taken from the north/south line. Mecca is also 31° more south, taken from the east/west line. Construct these angles in your drawing.
5. Draw a line CD parallel to the north/south line and a line AB parallel to the east/west line. Their point of intersection is F. OF depicts the direction to Mecca from Amsterdam.

Assignments

1. Use your graduated arc or your calculator to decide the angle between OF and the north/south line in this Arabic construction.
2. The coördinates of Havana are 23° N 82° W.
Decide upon the direction Havana/Mecca by using this Arabic construction.
3. The coördinates of Johannesburg are 26° S 28° E.
Decide upon the direction Johannesburg/Mecca by using this Arabic construction.

5. A notice, a cartoon and the Surinam Brethren (radio, January '91)



The contignents
are drifting
from each other

Wrong!

Caused by the globalness
of the earth they drift even so
much to each other in another
direction

In Surinam the Jawan Muslim Brethren Society has got two groups:
the 'East Brethren' and the 'West Brethren'.

The 'East Brethren' pray in eastern direction; the 'West Brethren' in west direction.

Both to Mecca.

6. The sun as a balloon over Mecca.

During the year the sun moves to and fro between the two tropics.

At the 21th of June she is over the northern tropic (23° N);

at the 22th of December she is over the southern tropic (23° S).

The coördinates of Mecca are about (21° N; 40° E).

Twice a year the sun is due over Mecca as a balloon. Why?

At what day (days) approximately will it happen?

Such a day it happens at 12 o' clock. Why?

At what day and at what o'clock can you look from your place on earth into the direction of the sun knowing it is the direction to Mecca? (time zones)

Will you be able to see the sun on your place on earth at that point of time?

7. 'Composed track': an alternating use of central projection and mercator projection.

Loxodrome and great circle

In navigation steering without changes in direction is preferred.

E.g. navigating via an almuqantar, heading remains 90^0 or 270^0 ; changes in direction are not necessary.

In general such a line on the globe with fixed course is called a 'loxodrome'.

0. Draw a straight line on the mercator map - why is that line a loxodrome on the globe?

However, a loxodrome between two points is generally not a great circle.

And therefore it is not the shortest distance between the points.

On the other hand via a great circle one must change the course continually, night and day.

Navigating officer

I ask a navigating officer. I ask him how he meets both requirements (a fixed course and the shortest distance) in practice.

He tells: "I use two maps: a central projection map (gnomic map) and a mercator projection map. If you have to sail e.g. from Land's End to New York across the Atlantic first I map out a straight line on the central projection between those two points. That straight line is a great circle.

The points of intersection with the meridians I copy on the mercator projection.

Then I connect these points with straight lines that are loxodromic on the mercator projection.

Thus, I navigate on a loxodromic line to the points of a great circle. We call this a 'composed track'"

Exercises

Use two maps: a mercator projection and a central (gnomic) projection of the North Atlantic.

Remember:

- a straight line on the mercator projection indicates a fixed course line on the globe (loxodrome).
- a straight line on the central projection indicates a great circle on the globe.

1. Draw a 'composed track' between Land's End and New York.
(Land's End is the most southwestern point of England)
2. What have you done? Step by step?
3. Columbus sailed from the Canarian Islands (La Gomera) to the Bahamas.
Draw a 'composed track' for such a trip.

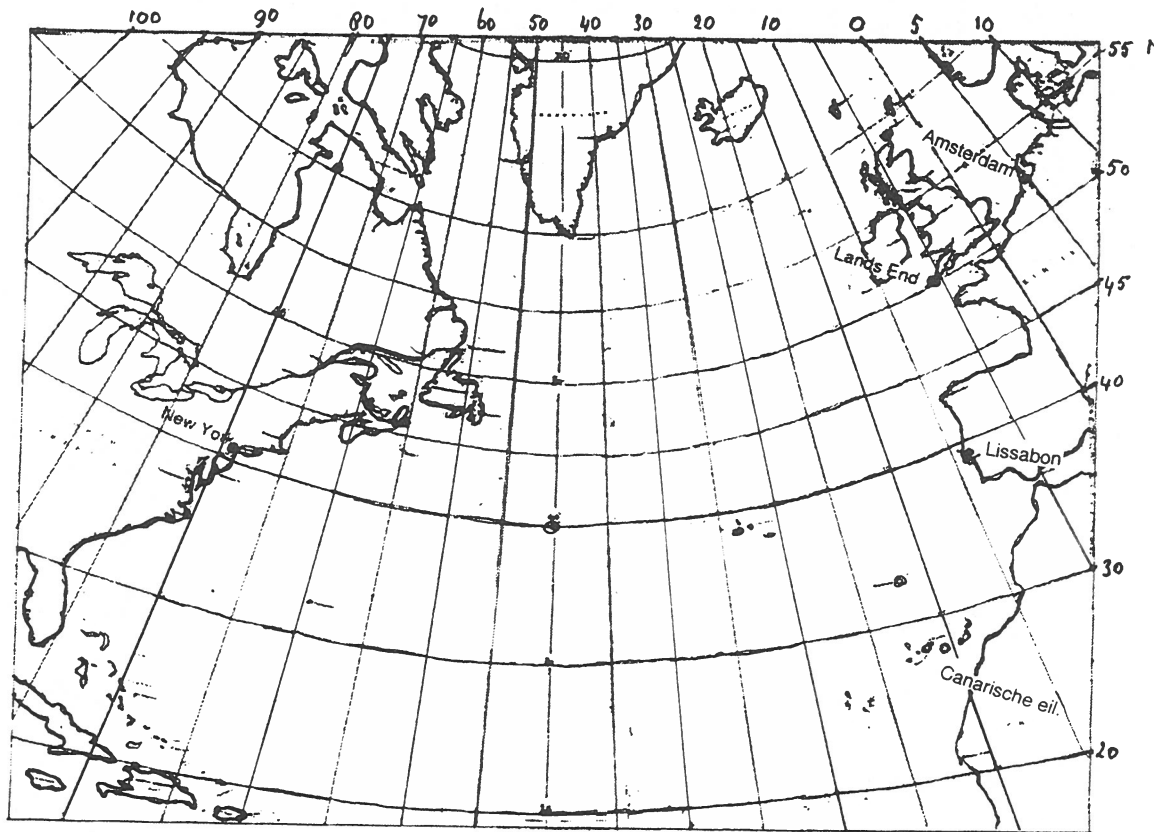
4. Draw a great circle track from Lisboa to New York on both a mercator projection and a central (gnomic) projection.
5. Does the shortest distance lead to the Azoren (a group of islands)?
6. What is the greatest latitude you reach on your trip from Lisboa to New York?
7. Which are the coordinates of that point?
8. Your great circle track will have points of intersection with the equator. What are the coordinates of these points?
9. What is the angle between both your great circle track and the equator?

10. What is the initial direction of a airplane flying from Amsterdam to Chicago via a great circle?

11. See the central projection map. Imagine this will be the global roof you see from the earth's centre. The distances increase going from the middle of the map to the margin. Which point is the middle?
12. What is the reason for the increasing distances?
13. The meridians are straight lines on the central projection. What are the almucantars? Circles? Demonstrate only the almucatar of 50° latitude is a parable.

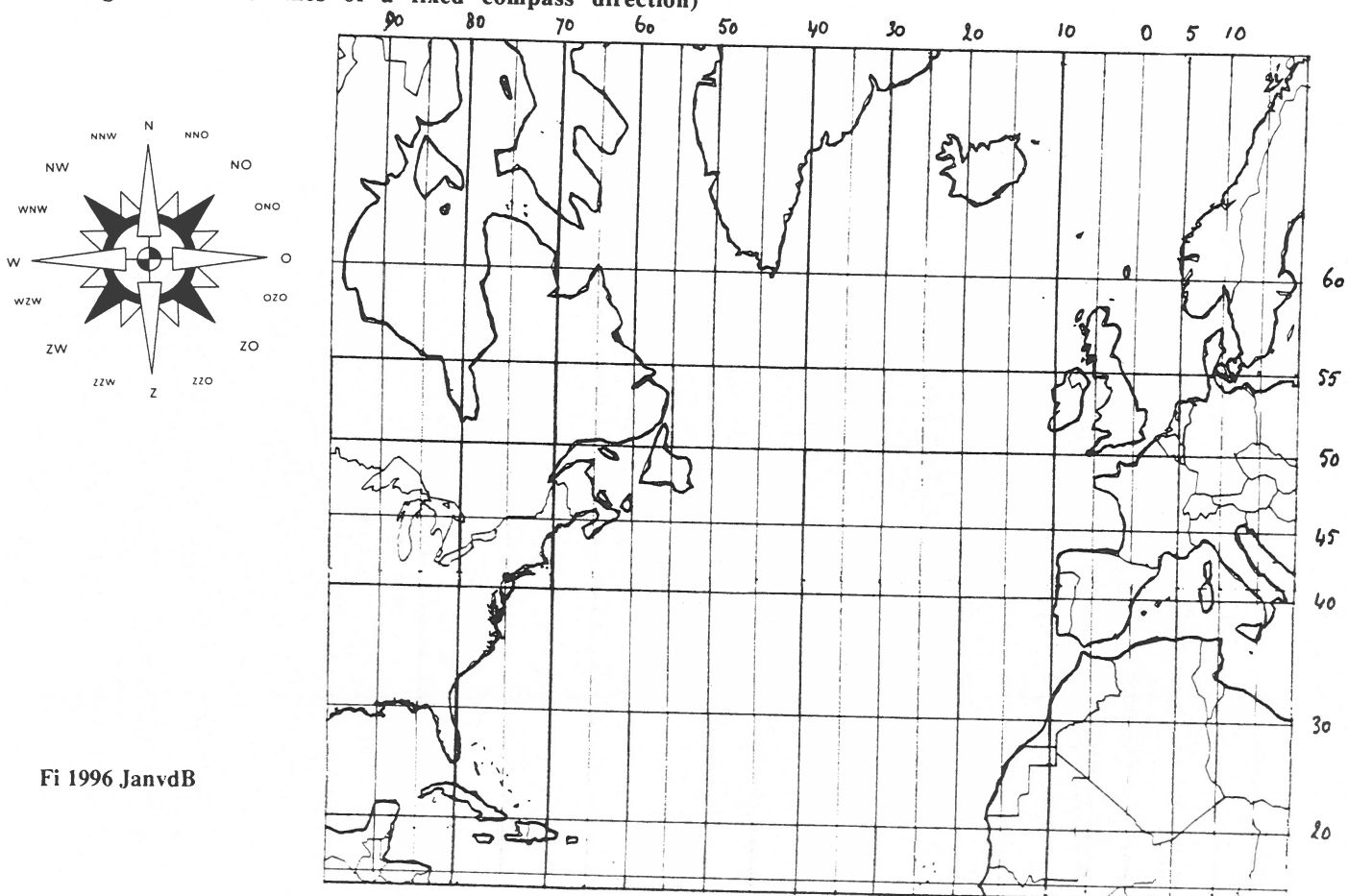
North Atlantic - Central projection (Gnomic projection)

(straigh lines are great circles, lines of the shortest distance on the earth surface)



North Atlantic - Mercator projection

(straight lines are lines of a fixed compass direction)



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Worksheets

1. Arrows to Mecca (chapter 1)
2. Airlines to draw (chapter 4)
3. Kibla map (chapter 4)
4. Compare solid geometry and spherical geometry

Press cuttings and other information about Mecca, to use in a project.

1. Boeing bound for Seoul (NRC, 16 Oct '90)
2. New mosque misses Mecca by three degrees (EC, 1 Nov '90)
3. Europe to Seoul: seven and a half hours faster (advertisement, 5 May '90)
4. Kibla (Encyclopaedia of Islam, Volume V, p. 84) (plus a description)
5. A notice, a cartoon and the Surinam Brethren (radio, Jan '91)
6. The sun as a balloon over Mecca
7. 'Composed track': an alternating use of central projection and mercator projection in navigation

Teacher's guide for Mecca

Level

Grade 3 (Pupils of the age of 16-17)

Number of lessons

Three or four

Material

- prayer mat
- globes
- strips of paper (or elastic bands, but then there is a danger of them shooting off)
- work sheets
- suggestions for projects

Collaboration with other subjects

Geography, History, World Perspectives, Language.

A great deal of language is involved: the story in the beginning, the newspaper report at the end.

One possible solution would be to do the reading classically or in small groups.

The text of chapter 1 may also serve as a reading assignment for English as subject.

Purport and aims

A discussion about where Mecca is situated leads to differentiating between representing the world by using a map or by using a globe.

The basic concept of this packet is that of the great circle : the circle drawn on the surface of the earth with its centre the centre of the earth.

It is the aim to come to an understanding of the following ideas:

1. the definition of the great circle as the biggest circle cutting the earth,
2. the great circle as the shortest distance between two points on the surface of the earth,
3. the great circle as a direction enabling one to get from one point on the earth's surface to another,
4. the shortest distance (great circle) between two points with the same latitude in the northern hemisphere rises "above" (to the north of) the circle representing their latitude (An aeroplane flying from Amsterdam to Los Angeles initially flies in an almost northerly direction),
5. travelling the shortest distance (great circle) between two points requires continuous adjustments to the direction with regard to the North Pole. (This is different from the situation when one travels between two points which lie close together: the destination can

then be reached without changing the direction of travel).

Connections with other mathematical topics and ideas

In the Mecca package different ideas and skills which students had acquired earlier occur again. Some of these are: locating position, computation and geometry, making sections, lines of vision. For example, in chapter 3 the line of vision to a (descending) balloon over Mecca features strongly. The line of vision lies in a "plane of vision" which bisects the earth as a great circle (because the balloon is descending towards the centre of the earth). The idea presented to the student is therefore that the direction in which Mecca lies, is given by a great circle.

Differentiation

The text is too difficult for weaker students. There are however ample opportunities for differentiation: involve the whole class when the background text is read; use globes and other materials extensively; grade the questions and discuss them beforehand; suggest different ways in which a project can be completed.

Answers and comments for the main chapters

1. Where is Mecca?

Comment

The discussion is initiated by posing two rhetorical questions. Students need not answer them; they serve to focus attention on the problem.

1a. -

1b. -

1c. -

Comment

The teacher may ask whether there are more positions on the surface of the earth from which will be looked in the same direction as from Havana. This idea features again later on when the great circle is introduced.

2. towards the east

Comment

The notation '300 Havana' reflects the Arabic order, whereas the 'English' custom is 'Havana 300'.

3. -

Comment

In this question the difference between a map and a globe of the earth can be introduced.

NB

'Direction' is not necessarily the direction of the shortest route - longer routes are also possible. However, 'direction' usually implies the direction for the shortest route.

4. the distance is then shorter

Comment

This can be observed on the globe with the use of strips of paper.

NB

Rubber bands are more difficult to handle and invite some pupils for shooting.

2. Great circles

5. Generally not

6. a. meridian circle

b. equator

c. parallel circle

Comment

Show which circles on the globe are called by which names

7. 2, "It is cutting the globe exactly in half"

8. 40.000 km
9. No, because that is the biggest circle
10. Two supplementaring meridians
11. -
- 12a. right (because the two halves are equal)
- 12b. right (there is no greater one)
- 12c. wrong (the two pieces do not necessarily have to be equal)
- 12d. wrong (the equator is a counter example)
- 12e. right (any biggest possible secant circle will cut the globe in half and thus go through the centre)
- 12f. right (imagine a cart with wheels of equal size on the left and on the right. It goes in a straight line around the earth. The centre of the earth will stay exactly under your cart).
- 12g. right (the circumference of the earth is 40.000 km)
- 12h. wrong (a circle, not parallel to the equator, can cut off a small section of the earth).

NB

Every great circle is not parallel to the equator, for : if parallel to the equator it isn't a greatcircle, but a parallel. So: property 12h is not sufficient for the definition of great circle)

13. No, since it is not the equator
14. At the earth's axis

Comment

This idea is very important for lateron in the book.

15. an endless amount
16. all of them
17. exactly one
- 18a. an endless amount
- 18b. yes
- 18c. at "the other side" of the earth

Comment

Question: "Can you be more precise?"

Answer 1: "There where the line Amsterdam/Centre of the earth comes out on the other side"

Answer 2: In coordinates: Amsterdam 52° north latitude 5° eastern longitude, then the antipole is 52° southern latitude 175° western longitude.

- 18d. yes
- 18e. yes
- 18f. yes (but for a lot of students this is difficult to grasp; remember the meridian through Amsterdam)
- 19a. an endless amount
- 19b. yes

- 19c. on the other side of the earth : 21° southern latitude 140° western longitude.
- 19d. yes
- 19e. yes
- 19f. yes
- 19g. yes, but it will not go through the north or south pole
- 19h. yes
- 20. -

Comment

Rules should be formulated by the students themselves. In this way they will master the knowledge they acquired for themselves. For example the rule of one pupil : "Pole and antipole are always in different hemispheres."

- 21. on the great circle
- 22. on the great circle
- 23. on the line **AB**
- 24. in the centre of the earth
- 25. on the earth's axis
- 26. yes.

NB

The centre of the parallel circle is higher than the centre of the equator. Let the parallel circle turn around **AB** until it merges with the great circle through **AB**.

- 27. no
- 28. yes, because the antipole of **A** is on the line through **A** and the centre of the earth.
- 29. the shortest arc **AB** on the great circle.

Comment

Turn the drawing upside down.

- 30. you can go from **A** to **B** over any circle but for the shortest distance you take the great circle.

3. Great circle, direction and shortest distance.

- 31. yes, because the balloon is pulled down to that point.
- 32. Yes, because the balloon is going straight down, which means in the direction of the centre of the earth.
- 33. because the centre of the earth is in the intersection plane.
- 34. -
- 35. 'middle perpendicular' of **AB**
- 36. M_3 , upper part
- 37. M_3 , bottom part
- 38. the great circle through **A** and **B**
- 39. -

40. the straight line **AB**
41. yes, the straight line **AB** is part of the great circle through **A** and **B** (the great circle that you can imagine going around the earth).

Comment

In our research school (in May of 1993) three kinds of geometry were involved:

1. on the 3D earth cutted by secant circles,
 2. on the 2D surface of the earth with a greatcircle as a line straight forward (not a straight line!)
 3. the 2D tangent plane in point **A** with straight lines in it.
42. yes, around the earth in the other direction.
43. -

4. Great circles on a map

44. -

Note: Do this in class first.

45. -

Note: Also this could be done in class first.

46. Shift the centre of the parallel circle to the centre of the great circle.

Note: repetition of question 26 in chapter 2

47a. -

Note: Let pupils use globes.

47b. -

47c. -

Note: The line is a kind of wave.

47d. no, it is a straight line (about a meridian)

48. the airline will go south of the parallel circle

49. along the equator because the equator is a great circle, thus the shortest distance between the two points.

50. on the equator, or on a meridian

51. -

52. measure it on the Kibla map or on the globe : about 60°

53. yes

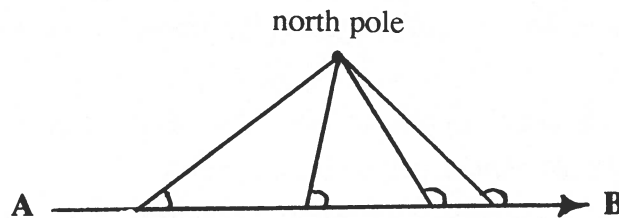
54. from there you can take any direction to Mecca

55. no. If he is on a great circle he will have to keep adjusting because the compass keeps direction according to the north pole. (If **A** and **B** are on a meridian or on the equator, adjusting is not necessary).

NB

This problem seems to contradict "our daily life on a flat plane". We aim for and go straight

to the point. But also on a flat plane we adjust if we keep direction according to a north pole:



We just don't notice it because usually we tacitly assume our "north pole" to coincide with our goal. In this way we can indeed go "straight to the point." A gyro-compass can realize such an artificial pole in practice.

56. yes, unless that great circle is the equator or a meridian.
 57. yes, the direction in relation to the north pole is changing.
 N.B.

This is a trick question : one does not consider the direction on the map, but on the globe meridians

58. meridians
 59. because of the changing angle between the route and the meridians.
 60. the projection is executed from the centre of the earth. Amsterdam, Los Angeles and the centre of the earth are in one secant plane. The great circle through Amsterdam and Los Angeles will thus be in that plane and is therefore a straight line on the polar map.
 Note: the polar map is a central projection.

61. Worksheet 4: Compare solid geometry and spherical geometry

**solid geometry
in the plane**

point in the plane
straight line

1. Infinite number of straight lines through a point

2. Only one straight line through two points

3. Two straight lines have one point in common or are parallel

**spherical geometry
on the sphere**

point on the sphere
great circle

1. Infinite number of great circles through a point

2. Only one great circle through two points or infinite if the points are antipoles

3. Two great circles have always two points in common

4. A point P outside straight line L.
Just one straight line
goes through P parallel to L

5. A point P outside straight line L.
Just one straight line
goes through P at right angle to L

6. Two points divide a straight line
into three parts

7. Only one of these three parts can be used
for defining distance of two points

8. There is no greatest distance between
two points

4. A point P outside great circle C.
Not any great circle
goes through P parallel to C

5. A point P.
There is a great circle C so that
infinite number great circles go through P
at right angle to C

6. Two points divide a great circle
into two parts

7. Both of these two parts can be used
for defining distance of two points

8. There is a greatest distance between
two points

Worksheets

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2. Airlines (chapter 4)
3. Kibla map (chapter 4)
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Press cuttings and other information about Mecca, to use in a project.

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7. 'Composed track': an alternating use of central projection and mercator projection.

Answers:

0. A straight line on the mercator map gives a loxodrome on the globe because the angles with the meridians are fixed.
1. -
2. -
3. -
4. -
5. No, it goes northerly
6. 44.5° N
7. 44.5° N 45° W
8. Note: (44.5° N, 45° W) was the biggest latitude. From 45° W 180° to the west and 180° to the east one finds the intersections: 225° W and 135° E
9. 44.5° (Note: the great circle is a secant circle)
10. Circa 292°
11. 40° N 45° W
12. Imagine the map as a tangent plane at point 40° N 45° W of the globe
13. The almucantars are conic sections.
Demonstrate only the almucatar of 50° latitude is a parabole by drawing the plane through the North pole, the center of the earth en the point 40° N 45° W.

7. 'Composed track': an alternating use of central projection and mercator projection.

Loxodrome and great circle

In navigation steering without changes in direction is preferred.

E.g. navigating via an almuqantar, heading remains 90^0 or 270^0 ; changes in direction are not necessary.

In general such a line on the globe with fixed course is called a 'loxodrome'.

0. Draw a straight line on the mercator map - why is that line a loxodrome on the globe?

However, a loxodrome between two points is generally not a great circle.

And therefore it is not the shortest distance between the points.

On the other hand via a great circle one must change the course continually, night and day.

Navigating officer

I ask a navigating officer. I ask him how he meets both requirements (a fixed course and the shortest distance) in practice.

He tells: "I use two maps: a central projection map (gnomic map) and a mercator projection map. If you have to sail e.g. from Land's End to New York across the Atlantic first I map out a straight line on the central projection between those two points. That straight line is a great circle.

The points of intersection with the meridians I copy on the mercator projection.

Then I connect these points with straight lines that are loxodromic on the mercator projection.

Thus, I navigate on a loxodromic line to the points of a great circle. We call this a 'composed track'"

Exercises

Use two maps: a mercator projection and a central (gnomic) projection of the North Atlantic.

Remember:

- a straight line on the mercator projection indicates a fixed course line on the globe (loxodrome).
- a straight line on the central projection indicates a great circle on the globe.

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(Land's End is the most southwestern point of England)
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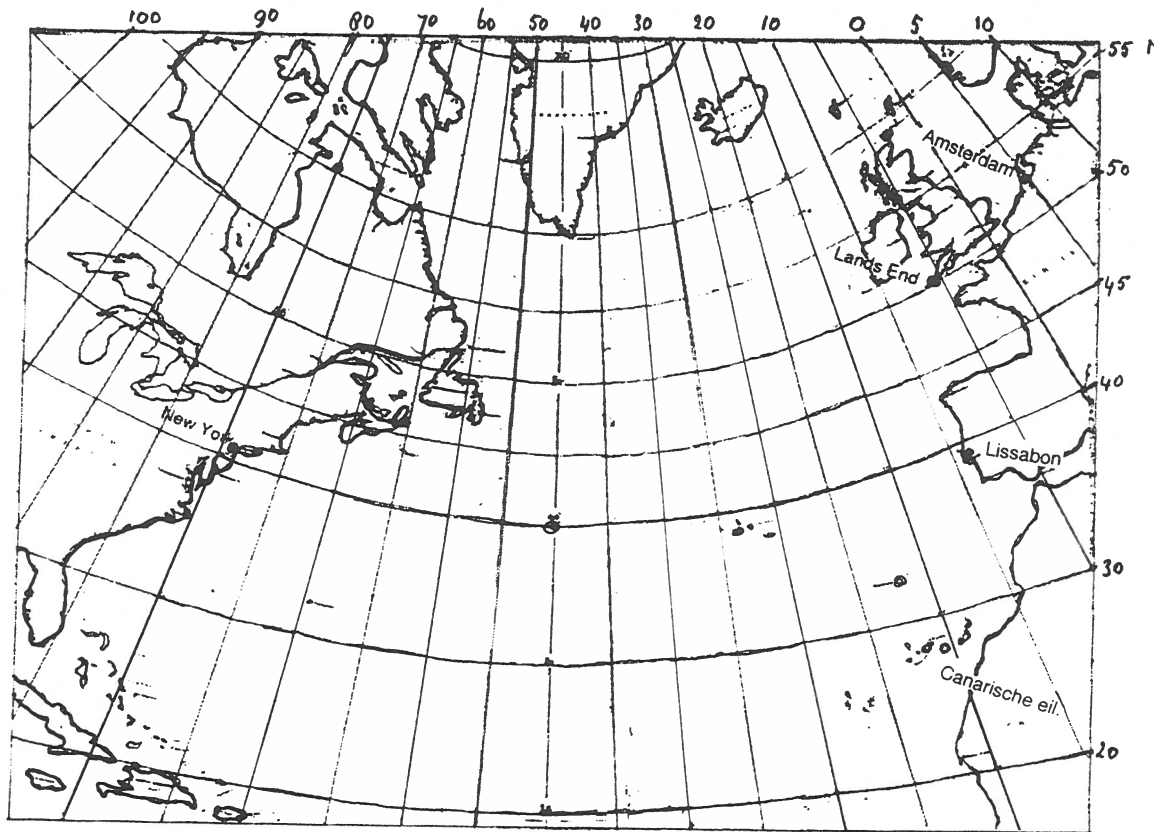
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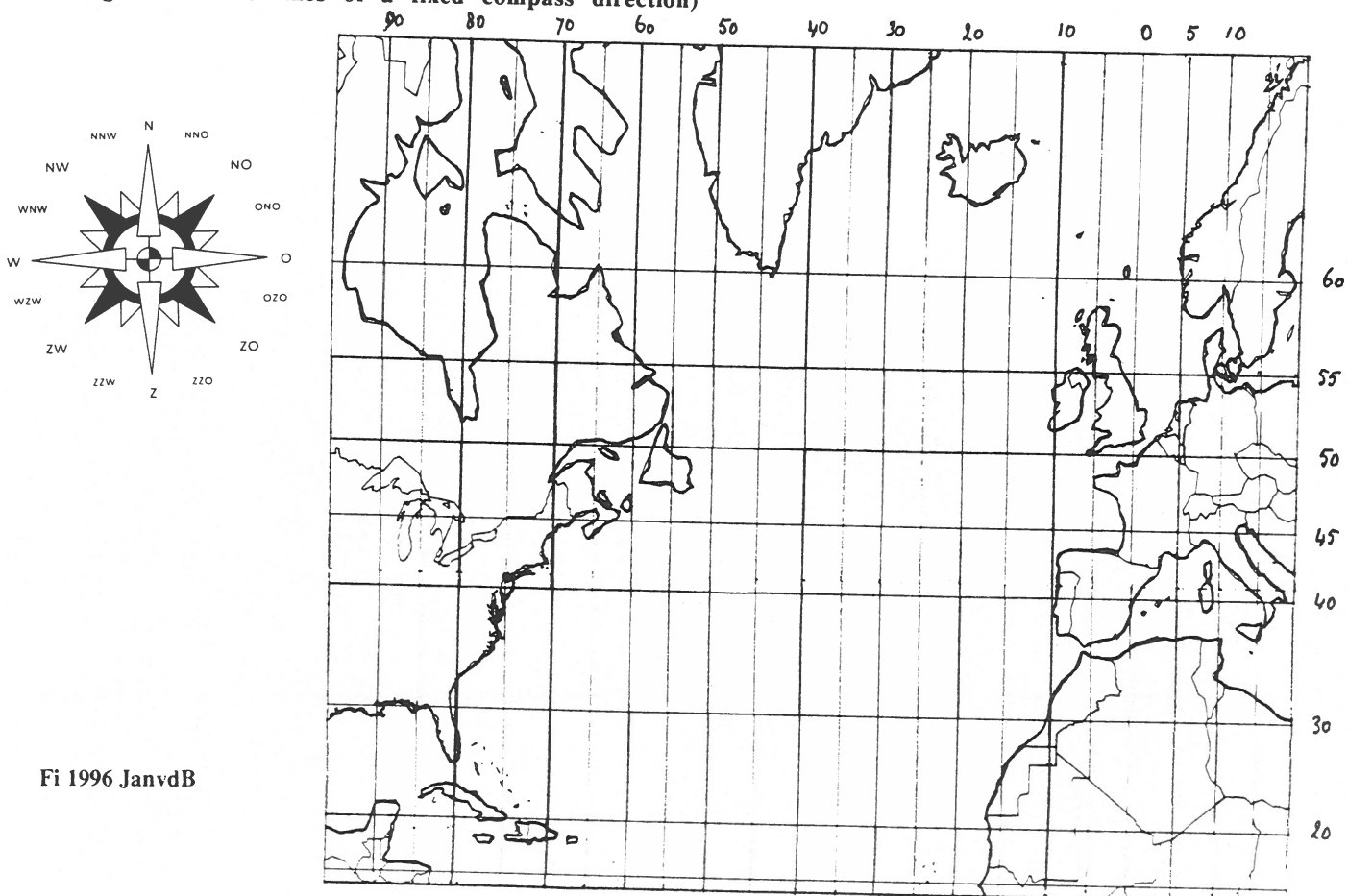
North Atlantic - Central projection (Gnomic projection)

(straigh lines are great circles, lines of the shortest distance on the earth surface)



North Atlantic - Mercator projection

(straight lines are lines of a fixed compass direction)



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Worksheets

1. Arrows to Mecca (chapter 1)
2. Airlines to draw (chapter 4)
3. Kibla map (chapter 4)
4. Compare solid geometry and spherical geometry

Press cuttings and other information about Mecca, to use in a project.

1. Boeing bound for Seoul (NRC, 16 Oct '90)
2. New mosque misses Mecca by three degrees (EC, 1 Nov '90)
3. Europe to Seoul: seven and a half hours faster (advertisement, 5 May '90)
4. Kibla (Encyclopaedia of Islam, Volume V, p. 84) (plus a description)
5. A notice, a cartoon and the Surinam Brethren (radio, Jan '91)
6. The sun as a balloon over Mecca
7. 'Composed track': an alternating use of central projection and mercator projection in navigation

Teacher's guide for Mecca

Level

Grade 3 (Pupils of the age of 16-17)

Number of lessons

Three or four

Material

- prayer mat
- globes
- strips of paper (or elastic bands, but then there is a danger of them shooting off)
- work sheets
- suggestions for projects

Collaboration with other subjects

Geography, History, World Perspectives, Language.

A great deal of language is involved: the story in the beginning, the newspaper report at the end.

One possible solution would be to do the reading classically or in small groups.

The text of chapter 1 may also serve as a reading assignment for English as subject.

Purport and aims

A discussion about where Mecca is situated leads to differentiating between representing the world by using a map or by using a globe.

The basic concept of this packet is that of the great circle : the circle drawn on the surface of the earth with its centre the centre of the earth.

It is the aim to come to an understanding of the following ideas:

1. the definition of the great circle as the biggest circle cutting the earth,
2. the great circle as the shortest distance between two points on the surface of the earth,
3. the great circle as a direction enabling one to get from one point on the earth's surface to another,
4. the shortest distance (great circle) between two points with the same latitude in the northern hemisphere rises "above" (to the north of) the circle representing their latitude (An aeroplane flying from Amsterdam to Los Angeles initially flies in an almost northerly direction),
5. travelling the shortest distance (great circle) between two points requires continuous adjustments to the direction with regard to the North Pole. (This is different from the situation when one travels between two points which lie close together: the destination can

then be reached without changing the direction of travel).

Connections with other mathematical topics and ideas

In the Mecca package different ideas and skills which students had acquired earlier occur again. Some of these are: locating position, computation and geometry, making sections, lines of vision. For example, in chapter 3 the line of vision to a (descending) balloon over Mecca features strongly. The line of vision lies in a "plane of vision" which bisects the earth as a great circle (because the balloon is descending towards the centre of the earth). The idea presented to the student is therefore that the direction in which Mecca lies, is given by a great circle.

Differentiation

The text is too difficult for weaker students. There are however ample opportunities for differentiation: involve the whole class when the background text is read; use globes and other materials extensively; grade the questions and discuss them beforehand; suggest different ways in which a project can be completed.

Answers and comments for the main chapters

1. Where is Mecca?

Comment

The discussion is initiated by posing two rhetorical questions. Students need not answer them; they serve to focus attention on the problem.

1a. -

1b. -

1c. -

Comment

The teacher may ask whether there are more positions on the surface of the earth from which will be looked in the same direction as from Havana. This idea features again later on when the great circle is introduced.

2. towards the east

Comment

The notation '300 Havana' reflects the Arabic order, whereas the 'English' custom is 'Havana 300'.

3. -

Comment

In this question the difference between a map and a globe of the earth can be introduced.

NB

'Direction' is not necessarily the direction of the shortest route - longer routes are also possible. However, 'direction' usually implies the direction for the shortest route.

4. the distance is then shorter

Comment

This can be observed on the globe with the use of strips of paper.

NB

Rubber bands are more difficult to handle and invite some pupils for shooting.

2. Great circles

5. Generally not

6. a. meridian circle

b. equator

c. parallel circle

Comment

Show which circles on the globe are called by which names

7. 2, "It is cutting the globe exactly in half"

8. 40.000 km
9. No, because that is the biggest circle
10. Two supplementaring meridians
11. -
- 12a. right (because the two halves are equal)
- 12b. right (there is no greater one)
- 12c. wrong (the two pieces do not necessarily have to be equal)
- 12d. wrong (the equator is a counter example)
- 12e. right (any biggest possible secant circle will cut the globe in half and thus go through the centre)
- 12f. right (imagine a cart with wheels of equal size on the left and on the right. It goes in a straight line around the earth. The centre of the earth will stay exactly under your cart).
- 12g. right (the circumference of the earth is 40.000 km)
- 12h. wrong (a circle, not parallel to the equator, can cut off a small section of the earth).

NB

Every great circle is not parallel to the equator, for : if parallel to the equator it isn't a greatcircle, but a parallel. So: property 12h is not sufficient for the definition of great circle)

13. No, since it is not the equator
14. At the earth's axis

Comment

This idea is very important for lateron in the book.

15. an endless amount
16. all of them
17. exactly one
- 18a. an endless amount
- 18b. yes
- 18c. at "the other side" of the earth

Comment

Question: "Can you be more precise?"

Answer 1: "There where the line Amsterdam/Centre of the earth comes out on the other side"

Answer 2: In coordinates: Amsterdam 52° north latitude 5° eastern longitude, then the antipole is 52° southern latitude 175° western longitude.

- 18d. yes
- 18e. yes
- 18f. yes (but for a lot of students this is difficult to grasp; remember the meridian through Amsterdam)
- 19a. an endless amount
- 19b. yes

- 19c. on the other side of the earth : 21° southern latitude 140° western longitude.
- 19d. yes
- 19e. yes
- 19f. yes
- 19g. yes, but it will not go through the north or south pole
- 19h. yes
- 20. -

Comment

Rules should be formulated by the students themselves. In this way they will master the knowledge they acquired for themselves. For example the rule of one pupil : "Pole and antipole are always in different hemispheres."

- 21. on the great circle
- 22. on the great circle
- 23. on the line **AB**
- 24. in the centre of the earth
- 25. on the earth's axis
- 26. yes.

NB

The centre of the parallel circle is higher than the centre of the equator. Let the parallel circle turn around **AB** until it merges with the great circle through **AB**.

- 27. no
- 28. yes, because the antipole of **A** is on the line through **A** and the centre of the earth.
- 29. the shortest arc **AB** on the great circle.

Comment

Turn the drawing upside down.

- 30. you can go from **A** to **B** over any circle but for the shortest distance you take the great circle.

3. Great circle, direction and shortest distance.

- 31. yes, because the balloon is pulled down to that point.
- 32. Yes, because the balloon is going straight down, which means in the direction of the centre of the earth.
- 33. because the centre of the earth is in the intersection plane.
- 34. -
- 35. 'middle perpendicular' of **AB**
- 36. M_3 , upper part
- 37. M_3 , bottom part
- 38. the great circle through **A** and **B**
- 39. -

4. A point P outside straight line L.
Just one straight line
goes through P parallel to L

5. A point P outside straight line L.
Just one straight line
goes through P at right angle to L

6. Two points divide a straight line
into three parts

7. Only one of these three parts can be used
for defining distance of two points

8. There is no greatest distance between
two points

4. A point P outside great circle C.
Not any great circle
goes through P parallel to C

5. A point P.
There is a great circle C so that
infinite number great circles go through P
at right angle to C

6. Two points divide a great circle
into two parts

7. Both of these two parts can be used
for defining distance of two points

8. There is a greatest distance between
two points

Worksheets

1. Arrows to Mecca (chapter 1)
2. Airlines (chapter 4)
3. Kibla map (chapter 4)
4. Compare solid geometry and spherical geometry

Press cuttings and other information about Mecca, to use in a project.

1. Boeing bound for Seoul (NRC, 16 Oct '90)
2. New mosque misses Mecca by three degrees (EC, 1 Nov '90)
3. Europe to Seoul : 72 hours faster (advertisement, 5 May '90)
4. Kibla (Encyclopaedia of Islam, Volume V, p.84)
5. A notice, a cartoon and the Surinam Brethren (radio, Jan '91)
6. The sun as a balloon over Mecca.
7. 'Composed track': an alternating use of central projection and mercator projection.

Answers:

0. A straight line on the mercator map gives a loxodrome on the globe because the angles with the meridians are fixed.
1. -
2. -
3. -
4. -
5. No, it goes northerly
6. 44.5° N
7. 44.5° N 45° W
8. Note: (44.5° N, 45° W) was the biggest latitude. From 45° W 180° to the west and 180° to the east one finds the intersections: 225° W and 135° E
9. 44.5° (Note: the great circle is a secant circle)
10. Circa 292°
11. 40° N 45° W
12. Imagine the map as a tangent plane at point 40° N 45° W of the globe
13. The almucantars are conic sections.
Demonstrate only the almucatar of 50° latitude is a parabole by drawing the plane through the North pole, the center of the earth en the point 40° N 45° W.

40. the straight line **AB**
41. yes, the straight line **AB** is part of the great circle through **A** and **B** (the great circle that you can imagine going around the earth).

Comment

In our research school (in May of 1993) three kinds of geometry were involved:

1. on the 3D earth cutted by secant circles,
 2. on the 2D surface of the earth with a greatcircle as a line straight forward (not a straight line!)
 3. the 2D tangent plane in point **A** with straight lines in it.
42. yes, around the earth in the other direction.
43. -

4. Great circles on a map

44. -

Note: Do this in class first.

45. -

Note: Also this could be done in class first.

46. Shift the centre of the parallel circle to the centre of the great circle.

Note: repetition of question 26 in chapter 2

47a. -

Note: Let pupils use globes.

47b. -

47c. -

Note: The line is a kind of wave.

47d. no, it is a straight line (about a meridian)

48. the airline will go south of the parallel circle

49. along the equator because the equator is a great circle, thus the shortest distance between the two points.

50. on the equator, òr on a meridian

51. -

52. measure it on the Kibla map or on the globe : about 60°

53. yes

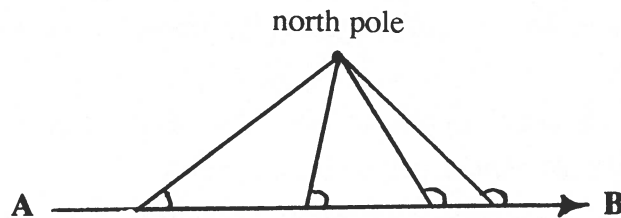
54. from there you can take any direction to Mecca

55. no. If he is on a great circle he will have to keep adjusting because the compass keeps direction according to the north pole. (If **A** and **B** are on a meridian or on the equator, adjusting is not necessary).

NB

This problem seems to contradict "our daily life on a flat plane". We aim for and go straight

to the point. But also on a flat plane we adjust if we keep direction according to a north pole:



We just don't notice it because usually we tacitly assume our "north pole" to coincide with our goal. In this way we can indeed go "straight to the point." A gyro-compass can realize such an artificial pole in practice.

56. yes, unless that great circle is the equator or a meridian.
 57. yes, the direction in relation to the north pole is changing.
 N.B.

This is a trick question : one does not consider the direction on the map, but on the globe meridians

58. meridians
 59. because of the changing angle between the route and the meridians.
 60. the projection is executed from the centre of the earth. Amsterdam, Los Angeles and the centre of the earth are in one secant plane. The great circle through Amsterdam and Los Angeles will thus be in that plane and is therefore a straight line on the polar map.
 Note: the polar map is a central projection.

61. Worksheet 4: Compare solid geometry and spherical geometry

**solid geometry
in the plane**

point in the plane
straight line

1. Infinite number of straight lines through a point

2. Only one straight line through two points

3. Two straight lines have one point in common or are parallel

**spherical geometry
on the sphere**

point on the sphere
great circle

1. Infinite number of great circles through a point

2. Only one great circle through two points or infinite if the points are antipoles

3. Two great circles have always two points in common

archief FI

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Mecca

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