DiaMech An instrument for Testing the understanding of Physics / Mechanics

by dr. Monica G.M. Ferguson-Hessler TUE (retired)

"We need to abolish teaching and begin to organise learning" (Ref.1)

DiaMech, the short name for **Diagnosis of Understanding of Mechanics,** has been developed to test students' understanding of fundamental concepts in the field of elementary Mechanics (point masses and translational motion) and the mastery of the relevant skills. The method can be used in all fields of Physics . This instrument does not test the reproduction of knowledge of laws and definitions, nor skills in problem solving.

DiaMech consists of this introductory chapter, and of four chapters containing the test units, as described in paragraph 2.4 below.

DiaMech Chapter 1

1.1. Objective

The objective of DiaMech is to offer teachers a possible new instrument to use within the digital teaching system they are using. DiaMech produces a profile of the student's understanding of fundamental mechanical concepts, indicating where her strong and weak points are, thus facilitating the planning and structuring of an individualized study course, or her ability to attend a collective course. (For ease of reading, we use "she" and "her" for all students and teachers!) The measurement can be repeated after any course to measure the change of the student's competence.

DiaMech is not an interactive learning environment but intended to be used by teachers in their own teaching systems. It does not demonstrate new phenomena, and it does not offer practice in problem solving.

The current DiaMech was developed for just one subject within Physics: Elementary Classical Mechanics. Looking at DiaMech from the perspective of Metacognition it is clear that the method can not only be used in all areas of Physics but also in other subjects where teaching aims to build understanding and skills for specific theoretical tasks. e.g. solving problems or making a diagnosis or a design.

1.2. How can we measure "understanding"?

In Physics, "understanding" means the ability to apply theoretical concepts and laws to a real-world situation to predict or calculate the development of some variable of the situation. For testing understanding, a system of Multiple True/False Test Statements is a good method as it can be used for testing for many concepts in many different situations.

1.2.1 The cognitive process of answering a True/False test.

Technically, DiaMech is a system of **Multiple True/False Test Statements.** But DiaMech asks for more than True/False choices: the student must also give **one argument** for her choice.

Answering a "Test Statement".

A Test Statement confronts students with a real-world situation, well described in a short text, and sometimes accompanied by a simple figure, e.g. “a girl is running over a hill at constant speed from A to B” (picture with a hill). Each "True/False" Test Statement refers to a specific element of the situation and its relation to physical concepts, e.g. “The girl is running with constant velocity all the way from A to B”. **The task of the student is to decide if the test statement is True or False, and to give a short argument for the choice**.

Looking at the cognitive processes that play a role in answering this type of questions, as in figure 1, we see that even a “True/False” choice requires a whole chain of processes. It starts with the **perception** of the situation described. In observing the situation and recognizing elements that are relevant for the statement; for this the student is guided by the content of her Long-Term Memory (LTM).

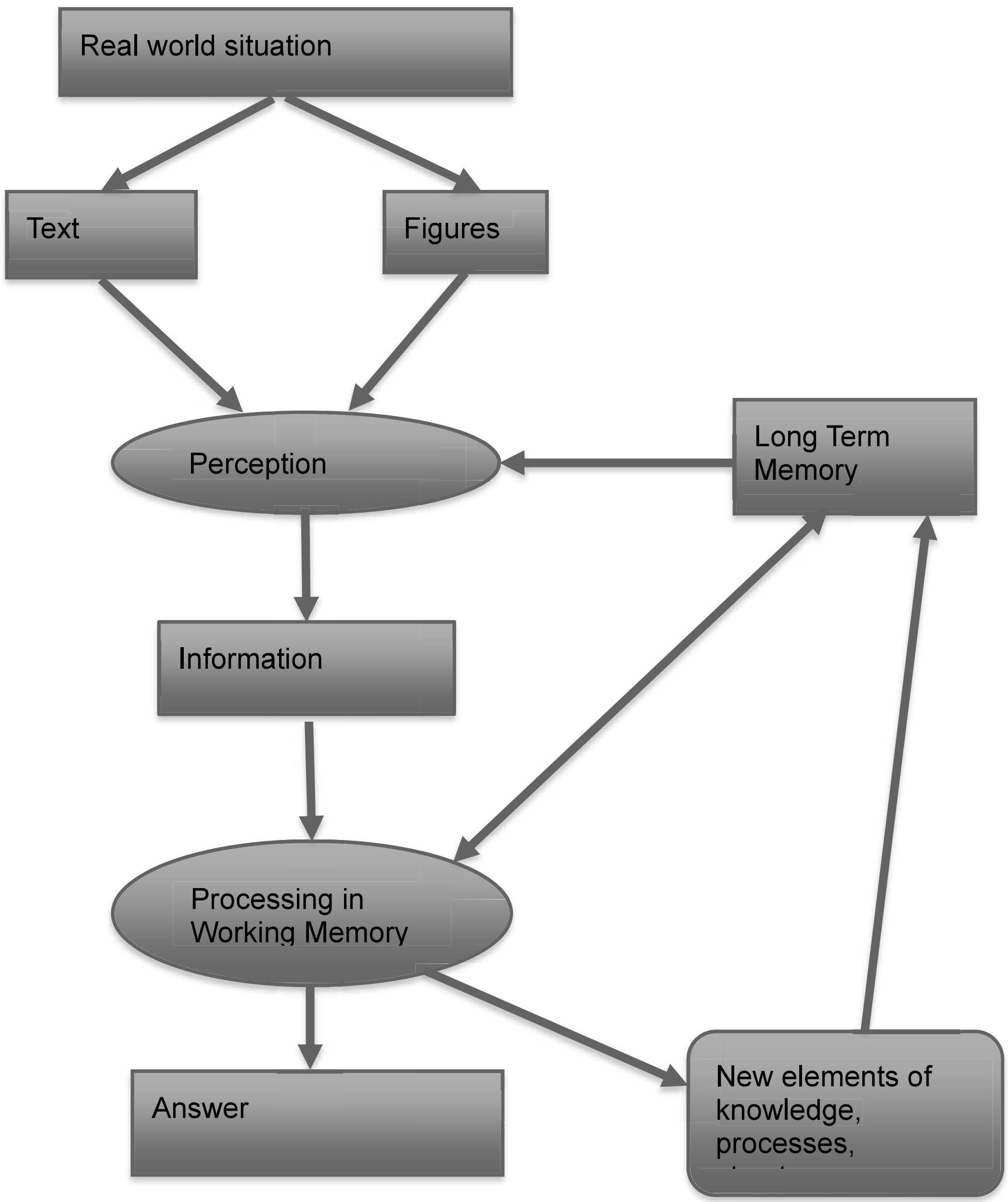
The **information** collected in the process of perception enters Working Memory (WM), where it is **interpreted**, physical laws and/or definitions are **applied** to the situation, and a **choice** is made between the answers True and False. This process is based on knowledge from LTM. In addition to the answer required, the whole process may lead to new elements of knowledge or processes, or new structures being added to LTM.

If the choice made is not correct, we can identify two points in the process of figure 1 where a mistake is possible:

1. If perception does not identify relevant information of the elements of the situation, this is due to lack of the necessary knowledge in Long Term Memory.
2. If processing in Working Memory (WM) is not successful, this is due either to lack of the necessary knowledge in WM, or to lack of skills in applying the knowledge to the situation given.

The student's reply to a True/False test statement of DiaMech does not distinguish between these situations. If an answer is incorrect, then either the student does not know the relevant definitions and laws, e.g. that *velocity is a quantity with a magnitude and a direction or* does not have the skill to interpret the situation, e.g. realizing that *running over a hill means that the vertical component of the velocity is not constant*. In both cases we conclude that the student does not know and/or understand the concept tested.

Figure 1. The process of answering a question in DiaMech.



1.2.3. Structure and content of the instrument

The **basic building block of DiaMech is the "Test Item",** consisting of a well-described simple real-world situation (as mentioned above), followed by 3 to 5 Test Statements, each relating some fundamental mechanical concepts to that situation, and requiring separate answers. For each Test Statement, DiaMech supplies two forms of Feedback, as explained in 2.5

DiaMech consists of a long series of such **Test Items,** covering the whole field of Elementary Classical Mechanics, as shown in Fig. 2. The test items are grouped into 4 chapters (see Fig. 3). each covering several concepts or laws.

Each chapter is divided into subchapters, each designed to test the understanding of one important definition or law. A subchapter starts with a short introduction for the teacher, specifying the content that is to be tested, and the notation used, and followed by the test items for that subject matter in the shape of a table with 3 – 5 statements.

To help teacher to find the relevant test units when they construct a test using DiaMech, the headers of each chapter explain in greater detail how the topics are arranged in subchapters.

1.2.4 The 4 chapters of DiaMech

After this introductory chapter, DiaMech starts with **Chapter 2**, a Chapter on **Graphs and Mathematics**. The reading of linear scales and two-dimensional diagrams, as for instance in Cartesian coordinates, is tested in several tasks. The mathematical part treats differentiation and integration, the concept of vector and its representations, addition, subtraction and scalar multiplication of vectors.

Chapter 3 is devoted to **Kinematics** (the description of the movement of a mass), starting with the concepts of position, velocity and acceleration and their application to different systems. It goes on to the interpretation of kinematical diagrams of position versus time, velocity versus time and acceleration versus time.

Chapter 4 contains applications of **Newton's three laws** . These applications do not require calculations but insight in the relation between forces acting on a system and the resulting motion of the system.

In **Chapter 5** the **conservation laws are applied** to mechanical energy. Again, applications do not test the skill in performing calculations, but the understanding of the different forms of mechanical energy, and their conservation and dissipation.

1.2.5. Using the DiaMech test system within your teaching environment

DiaMech is not intended for use as a stand-alone application, but as a method that teachers can incorporate into their own teaching environments.

- **For creating a test,** DiaMech offers the teacher a choice from usually three to five parallel **test items** (A to E), often referring to different situations but testing the same concept or skill. This means that DiaMech can be used for a first test, and then again, using different Test Items, for retesting students to measure the effect of some learning experience.

. **Example Test Unit Table**

Example: Properties of the derivative

Test Statement	***T/F***	***Because***

1. If a function f(x) is decreasing everywhere in the interval $x_1 < x < x_2$, then its derivative f'(x) has no zero in that interval.	***T***	*1. f'(x) is negative everywhere in this interval.*
2. If the derivative f'(x) of a function f(x) is known, then f(x) is also known	***F***	*2. Even if we know one correct f(x) an arbitrary constant can be added*

- To incorporate a particular test item into the questionnaire to be given to the students, **first copy the header of the test unit table, including the figure if present**
- Then **copy the test statements** in the rows of the table, followed on each row by two blank spaces to be filled in by the student. The first for her reply to the True/False question, the second for her short **"because"** statement justifying that choice.
- **Repeat that process** for all the test units to be included in the test.

1.2.6. Using feedback

For each test statement, DiaMech gives two types of feedback which the teacher can use.

- A **Short Feedback.** This is the "Correct / Not Correct" reply to the true/false answer given by the student,
- The "**Content Feedback**" is the **"because" statement from column 3 of the Test Item**. If the student gave the Right reply in column 2, that feedback will reinforce her choice. If she gave the Wrong reply, the correct "because" reply gives her a chance to understand her error.

Ideally, one would desire a "Full Feedback", including not only information on interpretation of the situation the correctness of the answer but also an explanation specifying why the answer the student gave is correct or incorrect.Teachers will very rarely have time to give personal feedback to every student's "because" answers, as each DiaMech reply contains many of these answers for each student But perhaps it will one day be feasible to build AI systems to create such feedbacks.

The teacher has the choice to give feedback in any form and at any time.

If the teacher gives the Content Feedback the students, have an immediate **learning experience.** The student with a correct choice can strengthen and widen her knowledge base and clear up points where she might have been hesitating. The student who has given incorrect arguments is confronted with her misunderstanding of the physical law used or of her lack of knowledge.

1.2.7. Diagnosis

DiaMech is designed to give a diagnosis of the student's state of knowledge of Mechanics, pinpointing:

- Lack of understanding of fundamental concepts, e.g. force, acceleration.
- Lack of understanding of fundamental laws, e.g. Newton's 2^{nd} law .

- Lack of skills in applying fundamental concepts, e.g. distinguishing forces acting on a system from forces acting within the system.
- Lack of problem-solving skills, e.g. defining a system, identifying forces acting on the chosen system, drawing a free-body diagram
- Lack of skills in the use of graphical representations: interpreting a given graph, deriving a new graph from a given one, qualitatively stating how a graph is going to change with a given change of the situation.
- Misconceptions, e.g. “if an object is at rest, then no forces act on it”.
- Lack of accurate observation of physical phenomena.
- Or the student is simply guessing!

This profile can be used to decide if the student has the prior knowledge required to enter a certain course. If that is not the case, then an individual study program can be designed. Designing this program would be the task of an experienced teacher. The following activities could make up part of such a study program:

- Reasoning with fundamental concepts to deepen understanding.
- Practice of applying fundamental concepts in a variety of situations.
- Practice of the analysis of problem situations
- Practice of graphical skills.
- Confrontation with experiences that stimulate abandoning misconceptions, and the construction of correct conceptions.
- Experience in the observation of several simple but fundamental mechanical phenomena.

1.2.8 Let students work in groups

20 years ago, when DiaMech was developed, teaching was often seen as bringing knowledge from the teacher/expert to the student. Today we know that the student's **own** learning activities are as important as the interactions with the teacher. With the increasing use of digital teaching, learning has more and more become **an individual activity,** a situation that is known to lead to personal stress for students**.** Meanwhile, research has shown that **interaction with other learners is one of the most important forms of learning.** Combining digital teaching using DiaMech with working in small groups offers a method for organizing student learning in a way that also leads to less personal stress.

One option. After a DiaMech test, give the students their Short Feedbacks, and divide them into groups of 3, if possible combining students with correct and incorrect answers. The groups are placed at separate tables in an instruction room and asked to discuss their own answers. This means asking questions, explaining difficult concepts and situations, discussing problem points, drawing conclusions together. A competent teacher is present to help each of the groups which encounter problems, not to give answers but to indicate the next step, offering the students the pleasure of discovering the answers themselves. At a later moment, give them the broad feedback as a check.

These group activities are exactly the activities that help students to strengthen their knowledge base, to discover their own misconceptions and, as a result stimulate their self-confidence.

1.3. Conclusion

Figure 2 shows the structure of the knowledge of Mechanics tested by DiaMech. Figure 3 shows the structure of DiaMech superimposed on the content, to make it easy for the teacher to find the relevant statements for a test.

Knowing the structure of the knowledge to be taught and learned is essential for everybody to avoid ending up with a bag of puzzle pieces instead of a structured knowledge basis.

In the constructive feedback, testing is combined with teaching.

- If the feedback is treated in small groups of students, this effect is strengthened.
- DiaMech is easily combined with automated task selection.

In all, we think that DiaMech can contribute to a motivating teaching environment with optimal use of digital teaching methods combined with personal contacts and so create a learning environment with less personal stress.

Figure 2. Mechanics of point masses and translational motion

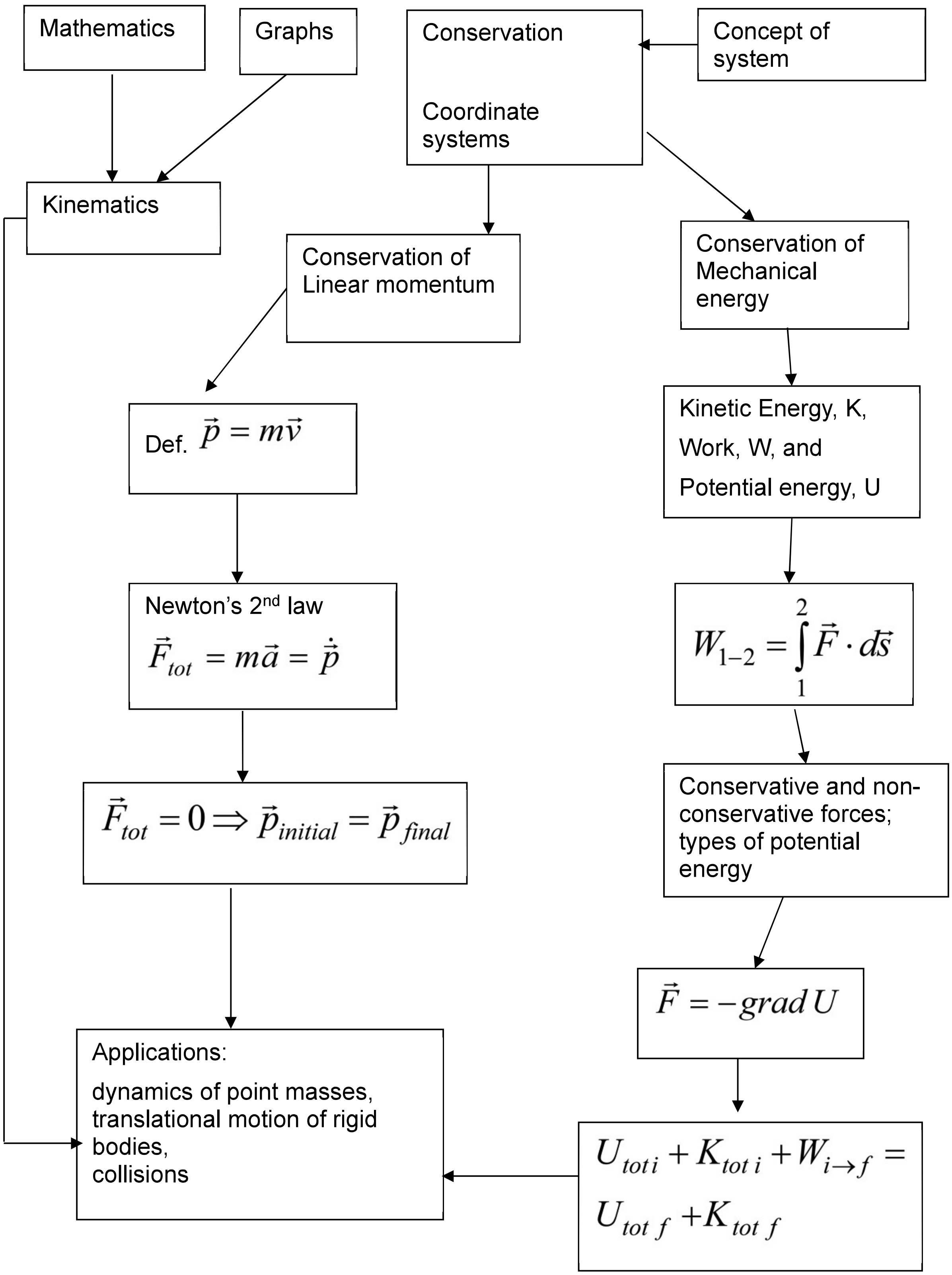
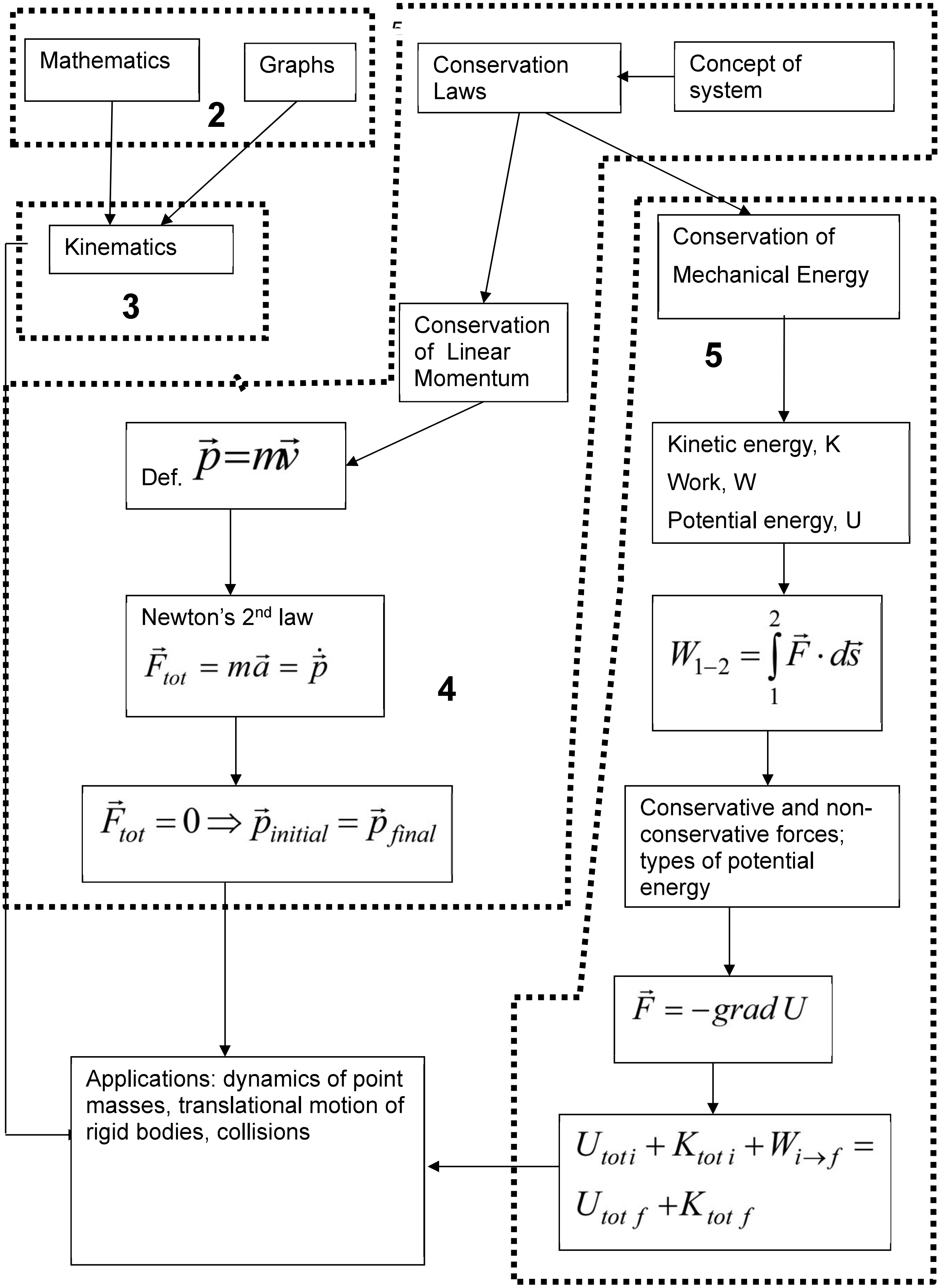


Figure 3. Chapters 2 to 5 of DiaMech address the indicated fields of Physics



With many thanks to my husband, Eric Ferguson, PhD,. for his assistance in making my Word documents from 20 years ago available for teachers and students of today.

Monica G.M. Ferguson-Hessler Ph.D.

Retired from Eindhoven University of Technology, Netherlands

m.ferguson@antenna.nl

The theoretical basis of the diagram in figure 1 is found in

Types and Qualities of Knowledge

T de Jong, MGM Ferguson-Hessler

Educational Psychologist **31,** 2 (1996)

Cited by 1354 related articles All 8 versions

DiaMech Chapter 2: Mathematics

Introduction

This is Chapter 2 of DiaMech, a test of Knowledge and Skills in Elementary Mechanics. Please read the introductory Chapter 1- before using this chapter.

Knowledge and skills tested:

Knowledge	Skills
1. Concepts of scales and graphs	1. Reading/interpreting a variety of scales and graphs
2. Cartesian coordinates	2. Determining positions, distances and directions
3. Concept of differentiation	3. Finding the derivatives of a variety of functions; relating the graphical representations of a function to its derivative
4. Concept of integration-	4. Integrating a variety of functions; relating the graphical representation of a function and its primitive
5. Concepts of vector and unit vector. - Representations of vectors as arrows or by components. Vector Algebra: addition, subtraction, scalar product.	5. Resolving vectors into components.- Translating between representations. - Performing vector algebra in both in both representations

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2.1. Concept of Scales

2.1.1. Linear Scales:

In a scale, numbers are represented by points on a line .That line may have any shape, but in many situations it is simplest to use a straight line. Other shapes are also common: a circular scale is very suited to representing directions in space, or the time on a clock.

If all unit steps are of the same physical size we speak of a linear scale.

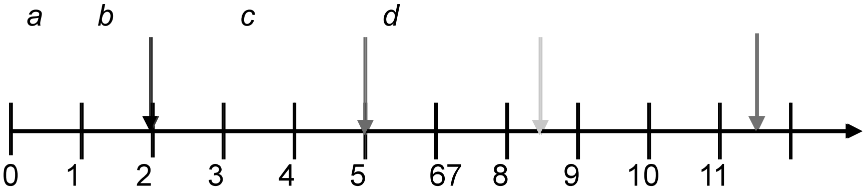
When reading from a scale, it is often necessary to interpolate between the values marked on the axis.

2.1.1. 1.Reading Linear scales

{Comment to Editor: change colour of arrows to black and fit letters to identify the arrows for tests A to E inclusive)

2.1.1.A Read the positions of the four arrows with an accuracy of 0.5:

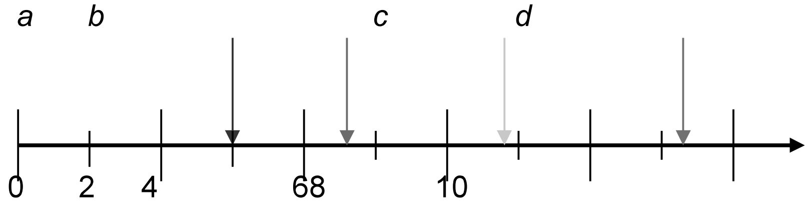
d *a* *b* *c* *d*



Answers: *a* 2, *b* 5, *c* 7.5, *d* 10.5

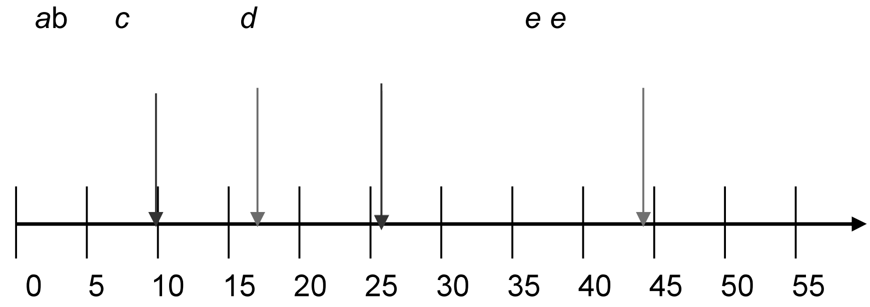
2.1.1. B. Read the position of the four arrows with an accuracy of 0.2

d *a* *b* *c* *d*



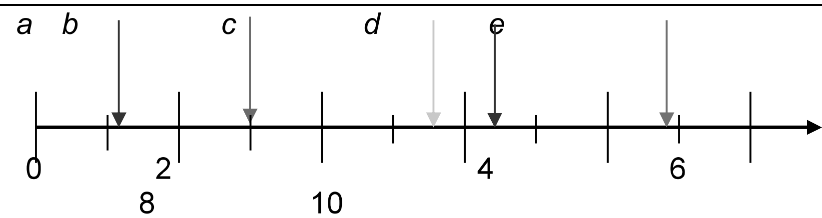
Answers: *a* 3.0, *b* 4.6, *c* 6.8 *d* 9.4

2.1.1 C. Read the position of the four arrows with an accuracy of 1.0



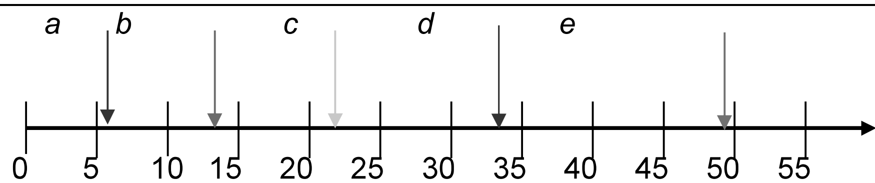
Answers: *a* 10, *b* 17, *c*26, *d*38, *e* 44

2.1.1. D. Read the position of the five arrows with an accuracy of 0.1



Answers: *a* 1.2, *b* 3.0 *c*5.6 *d* 6.4 *d* 8.8

2.1.1. E. Read the position of the five arrows with an accuracy of 1



Answers: *a* 6, *b*13, *c*22, *d* 33, *d* 49

2.1.2. Non-linear scales

In certain practical situation, linear scales, where every unit step occupies the same space, are unpractical. Sometimes the data may may cover a broad range of orders of magnitude, and a on a linear scale must of the data points would be bunched up near the origin. A circular scale could be very useful in representing directions on the ground or time on a clock. In this DiaMech chapter, these scales are not discussed, as they are hardly used in describing mechanics (without rotations) in a plane.

2.2. Cartesian Coordinates

DiaMech tests are limited to positions and motion in two dimensions, and omits rotational movements in a plane. Positions and movements in three dimensions, polar coordinates and rotating objects are not covered.

Cartesian coordinates use two perpendicular axes (usually called the *x-axis* and the *y-axis*). Any point *(X,Y)* on the plane can be precisely defines by giving the distances *X* (from the *y-axis)* and *Y* (from the *x-axis)* . Both coordinates can be positive or negative. Cartesian coordinates can be used for describing positions and movement in the plane.

In DiaMech we use only linear scales along the axes, but in other situations non-linear scales can also be used.

2.2.1. Reading graphs

The following tests cover the use of cartesian coordinates.

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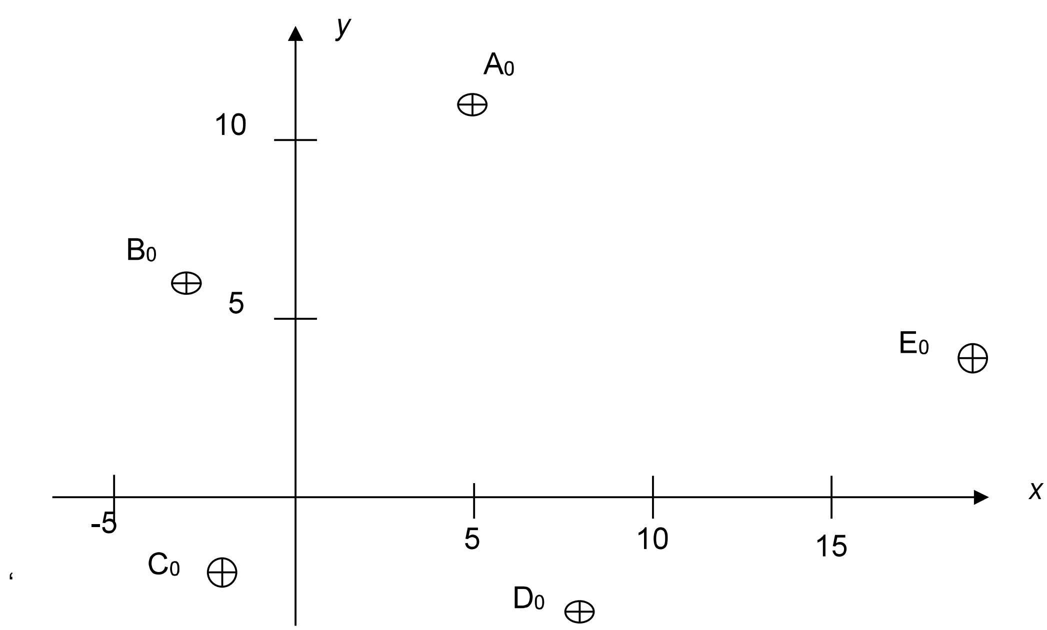
2.2.1.A In the graph below five points have been marked, A_0, B_0,.....E_0

Read the coordinates of each of the five points to an accuracy of 1.

{Comment to Editor: add coordinate squares for x= -5 to 15 and y= 5, 10, and similarly at the values marked on the axes on 2.2.1. B to E)



Answer: A_0: (5,11); B_0 (-3,6); C_0: (-2,-2); D_0: (8,-3); E_0: (19,4)

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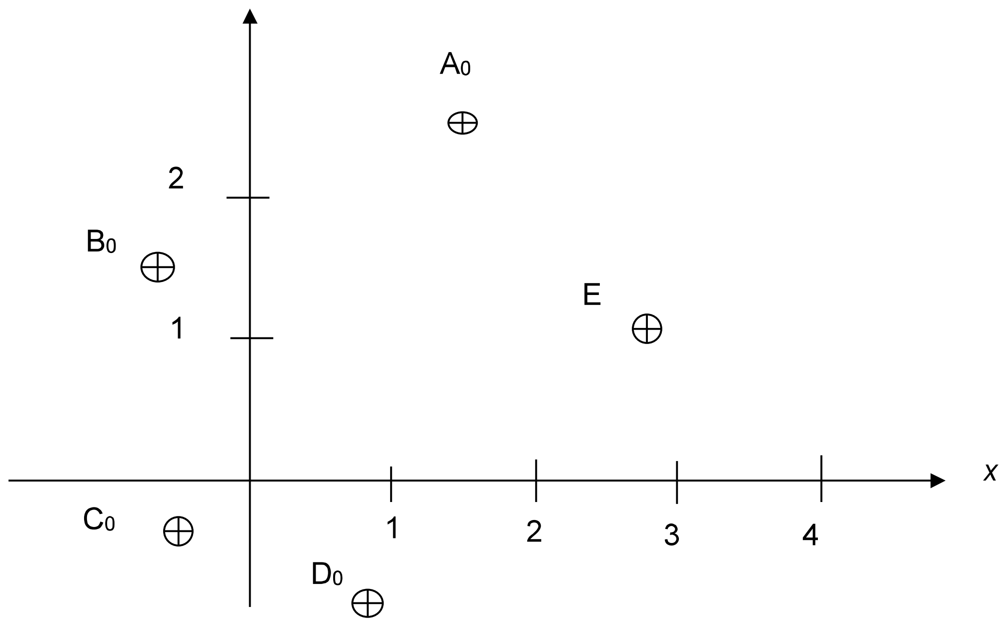
2.2.1.B .In the graph below five points have been marked, A_0, B_0,.....E_0

Read the coordinates of each of the five points to an accuracy of 0.1



Answer: A_0: (1.5, 2.5), B_0: (-0.7, 1.5), C_0: (-0.5, -0.3), D_0: (0.8, -0.8), E_0: (2.8, 1.1)

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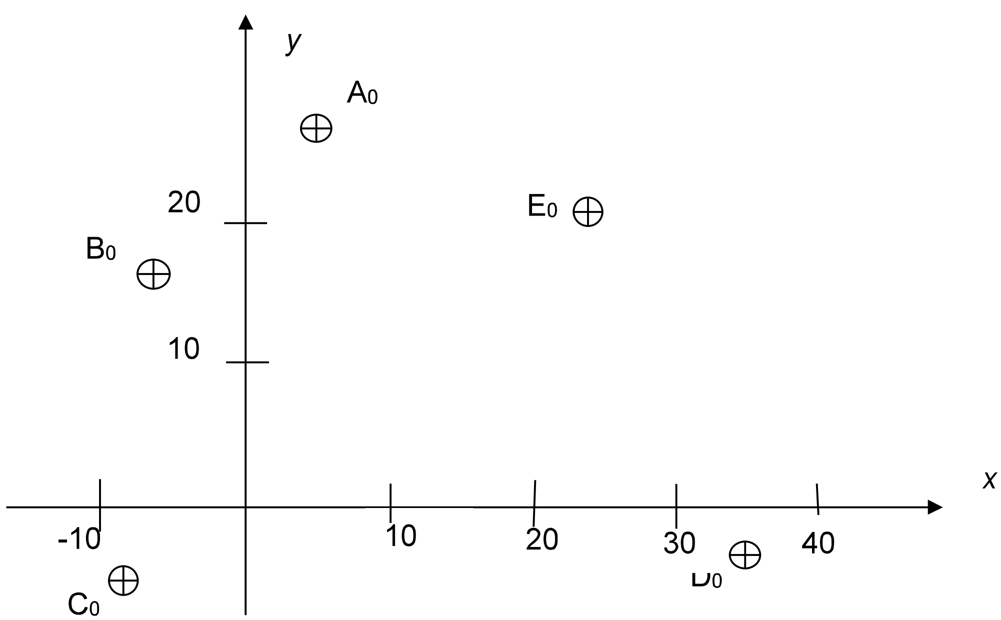
2.1.2.1.C. In the graph below five points have been marked, A_0, B_0,.....E_0

Read the coordinates of each of the five points to an accuracy of 1.



Answer: A_0: (5, 27), B_0: (-7, 17), C_0: (-8, -5), D_0: (35, -3), E_0: (24, 21).

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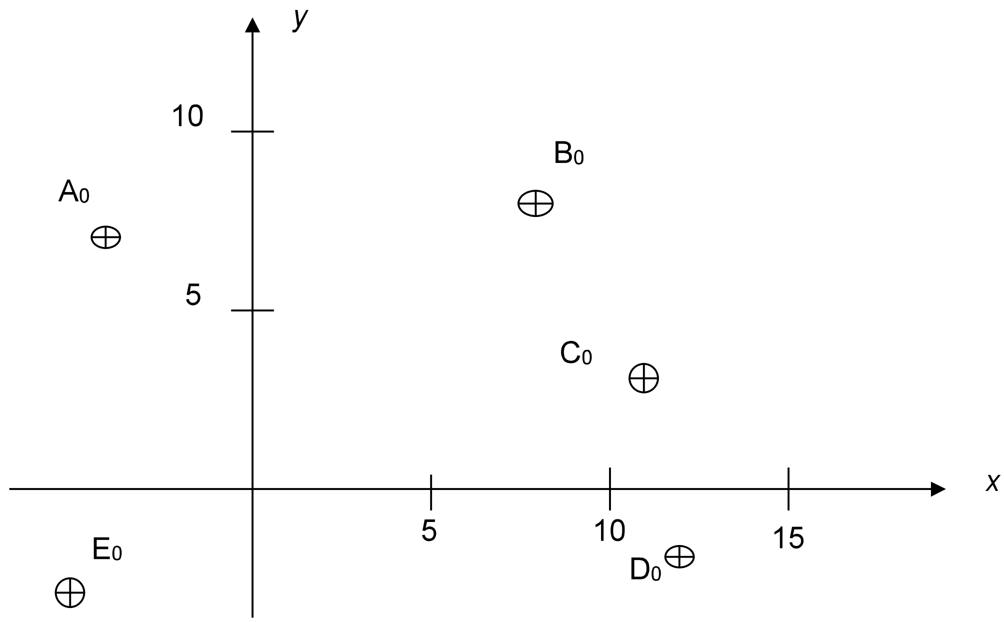
2.2.1.D. In the graph below five points have been marked, A_0, B_0,.....E_0

Read the coordinates of each of the five points to an accuracy of 1.



Answer: A_0: (-4,7), B_0: (8,8), C_0: (11,3), D_0:12, -2), E_0: (-5, -3).

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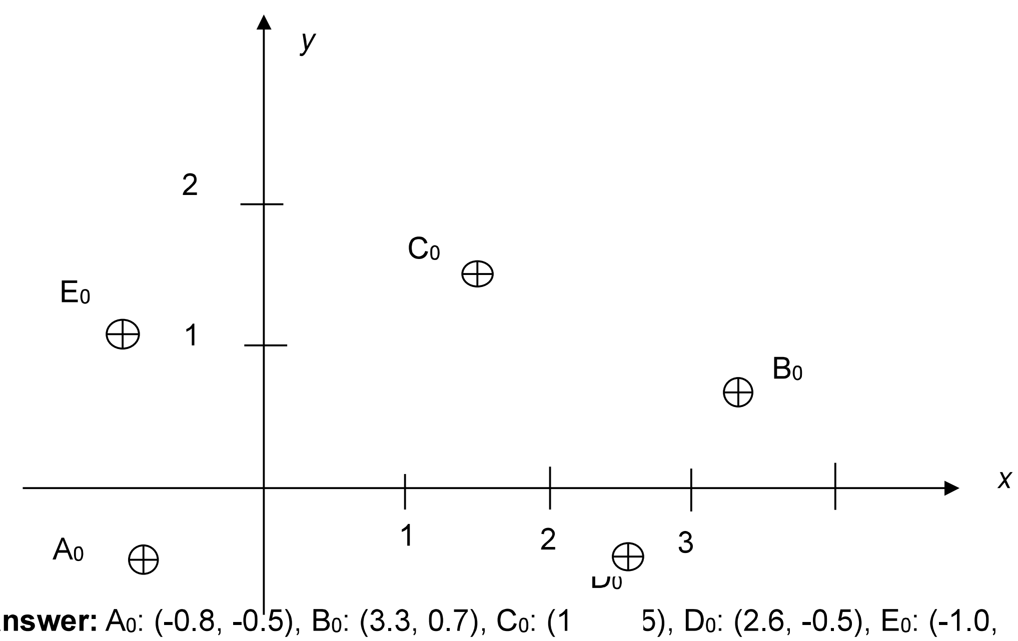
2.2.1.E. In the graph below five points have been marked, A_0, B_0,.....E_0

Read the coordinates of each of the five points to an accuracy of 0.1.



Answer: A_0: (-0.8, -0.5), B_0: (3.3, 0.7), C_0: (1 5), D_0: (2.6, -0.5), E_0: (-1.0, 1.2

++

2.3. Concept of differentiation

2.3.1. The definition of differentiation:

Differentiating a function $y = f(x)$ is the process of finding the **derivative** of f(x) $\frac{dy}{dx} = f'(x)$, defined by $f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$ . . . . (1)

where Δx is a small. change in x. This definition shows that the derivative of f(x) in punt x1 gives the slope of the function at this point.

All the questions below use only functions for which a derivative exists.

2.3.1.1. Properties of the derivative [see formula (1) above]

2.3.1.1.A

Test Statement	T/F	Because
1. If a function *f(x)* is decreasing everywhere in the interval $x_1 < x < x_2$, then its derivative f'*(x)* has no zero in that interval.	**T**	1. *f'(x)* is negative everywhere in this interval.
2. If the derivative *f'(x)* of a function *f(x)* is known, then *f(x)* is also known	**F**	2. Even if we know one correct *f(x)* an arbitrary constant can be added
3. If the derivative is zero for a certain $x=x_1$, then the function has a minimum for $x=x_1$	**F**	3. If the derivative is zero for $x=x_1$, then the function can also have a maximum

2.3.1.1.B

Test Statement	T/F	Because
1. If the function $f(x) > 0$ for all values of x in the interval $x_1 < x < x_2$, then the derivative cannot be negative in this interval	**F**	1. In the interval *f(x)* can both increase and decrease, so the derivative can be both positive and negative
2.A positive value of the derivative *f'(x)* at $x = x_1$ indicates that the function *f(x)* increases with increasing *x* at that point	**T**	2. The positive derivative means that the function is increasing at that point
3. If $x_1 < x < x_2$, $f'(x_1) > 0$ and $f'(x_2) < 0$, then the function *f(x)* has at least one maximum in the interval	**T**	3. A change in the sign of *f'(x)* from positive to negative indicates a maximum of the function *f(x)*
4. The definition shows that the derivative of *f(x)* in any punt gives the slope of the line at that point	**T**	4. Indeed

2.3.1.1.C

Test Statement	T/F	Because
1. If the function *f(x)* has a zero in the interval $x_1 < x < x_2$, then its derivative *f'(x)* changes sign somewhere in that interval	**F**	1. A zero of the function does not imply a zero of the derivative
2. If the derivative *f'(x)* is negative for all the interval $x_1 < x < x_2$, then the function *f(x)* changes from positive to negative in that interval.	**F**	2. A negative derivative indicates that the value of the function decreases all through the interval, but the function need not change sign.
3 .If the function *f(x)* has a maximum for $x = x_1$, then the derivative *f'(x)* is zero for $x = x_1$,	**T**	3. When the function has a maximum the derivative must be zero

2.3.1.1.D

Test Statement	T/F	Because
1. If *f(x)* is constant and positive everywhere in the interval $x_1 < x < x_2$, then the derivative *f'(x)* is also constant and positive in that interval	**F**	1. If the function is constant, then its derivative is zero everywhere
2. If the derivative *f'(x)* of the function *f(x)* is positive everywhere in the interval $x_1 < x < x_2$, then *f(x)* is increasing in every point of that interval	**T**	2. A positive derivative indicates that the value of the function increases everywhere in the interval
3. If *f(x)* has one maximum and one minimum for $x_1 < x < x_2$, then the sign of $f'(x_1)$ differs from the sign of $f'(x_2)$	**F**	3. The derivative changes sign at each maximum and each minimum. The sign changes twice, and is the same at beginning and end.. .

2.3.1.1.E

Test Statement	T/F	Because
1. If the function *f(x)* is increasing for $x < x_1$ and decreasing for $x > x_1$, then $f'(x_1) = 0$	**T**	1. *f(x)* has a maximum for $x = x_1$, so its derivative is zero at that point.
2. If the derivative *f'(x)* of the function *f(x)* has a constant value everywhere in the interval $x_1 < x < x_2$, then *f(x)* is the equation of a straight line	**T**	2. If the derivative is constant, then the function is a linear function of *x*
3. If the derivative *f'(x)* of the function *f(x)* has a minimum at $x = x_1$, then $f(x_1) = 0$	**F**	3. A minimum in the derivative *f'(x)* gives no information on the value of *f(x)*. .

2.4. The concept of integration

2.4.1. The definition of integration:

Integration of a function *g(x)* is the process of finding a **primitive function** *G(x)*of *g(x)*, which has the property *G'(x) = g(x).* In other words, *G(x)* is a function which has the given function *g(x)* as a derivative. Of course any function *G(x) + K,* , where *K* is a constant, is also a primitive of *g(x)*.

Another way of expressing integration is that the change in value of the primitive *G(b) – G(a)* on any interval *a<x< b* always equals the value of the definite integral $\int_a^b g(x)dx$.

2.4.1.1. Properties of the primitive function

2.4.1.1.A

Test Statement	T/F	Because
1. If an algebraic function *g(x)* is given, there is always a unique primitive function *G(x)* of *g(x)*.	**F**	1. There is no unique primitive; If one adds a constant to *G(x)* the derivative remains the same.
2. If function $G_1(x)$ has the derivative $g_1(x)$ and $G_2(x)$ has the derivative $g_2(x)$, then $g_1(x) + g_2(x)$ is the derivative of $G_1(x) + G_2(x)$	**T**	2. The derivative of the sum is the sum of the derivatives.
3. The definite integral of a function *g(x)* between the limits *a* and *b* is not uniquely defined, but a constant can always be added	**F**	3. the definite integral between *a* and *b* is uniquely defined

2.4.1.1.B

Test Statement	T/F	Because
1. If *G(x)* is a primitive function to both $g_1(x)$ and $g_2(x)$, then the functions $g_1(x)$ and $g_2(x)$ are identical.	**T**	1. The derivative of a given function is uniquely defined.
2. If *g(x)* is the derivative of a function *G(x)* then *{g(x)+a}* is also a derivative of *G(x)*.	**F**	2. The derivative is uniquely defined once the function is given.
3. The graphical interpretation of the definite integral of *g(x)* on the interval *a<x<b* is the area limited by the x-axis, the lines x = a and *x=b* and the curve *g(x).*	**T**	3. This is precisely the definition of a definite integral.

1.4.1.1. C

Test Statement	T/F	Because
1. If *G(x)* is a primitive function of *g(x)*, and *c* is a constant, then *{G(x)+c}* is also a primitive function of *g(x)*.	**T**	1. The additive constant *a* does not change the derivative
2. If $G_1(x)$ and $G_2(x)$ are both primitive functions of *g(x)*, then $G_1(x)$ and $G_2(x)$ are identical	**F**	2. $G_1(x)$ and $G_2(x)$ may differ by a constant and still have the same derivative.
3. If $\int_a^b g(x)dx = 0$, then *g(x) = 0* for all values of *x* in the interval $a \leq x \leq b$.	**F**	3. If *g(x)* takes on both positive and negative values in the interval, then the integral may equal zero.

2.4.1.1. D

Test Statement	T/F	Because
1. If $G_1(x)$ and $G_2(x)$ are both found to be a primitive function of *g(x),* then $G_1(x) = G_2(x)$	**F**	1. $G_1(x)$ and $G_2(x)$ may still differ by a constant
2. If g*(x)* is a polynomial of degree *n*. then its primitive function is a polynomial of degree *n+1*.	**T**	2. The derivative of x^n is nx^{n-1} .
3. If *G(x)* is the primitive function of *g(x)* and *G(a) = G(b)* then $\int_a^b g(x)\,dx = 0$	**T**	3. This follows directly from the definition of the definite integral

2.4.1.1.E

Test Statement	T/F	Because
1. Two functions $g_1(x)$ and $g_2(x)$ have primitive functions $G_1(x)$ and $G_2(x)$ respectively. Then $[G_1(x) + G_2(x)]$ is the unique primitive function of $[g_1(x) + g_2(x)]$	**F**	1. A constant may still be added to $[G_1(x) + G_2(x)]$ so this primitive function is not unique.
2. If a polynomial g*(x)* is given, it is always possible to find a polynomial G*(x)*, which is a primitive function of g*(x)*.	**T**	2. For a polynomial a primitive can easily be derived.
3. If $\int_a^b g(x)\,dx < 0$, where $a < b$, then $g(x)<0$ for all values of *x* in the interval $a \leq x \leq b$.	**F**	3. *g(x)* must be negative for some values of *x*, but it may also be positive for some values of *x*.

2.5. Vectors

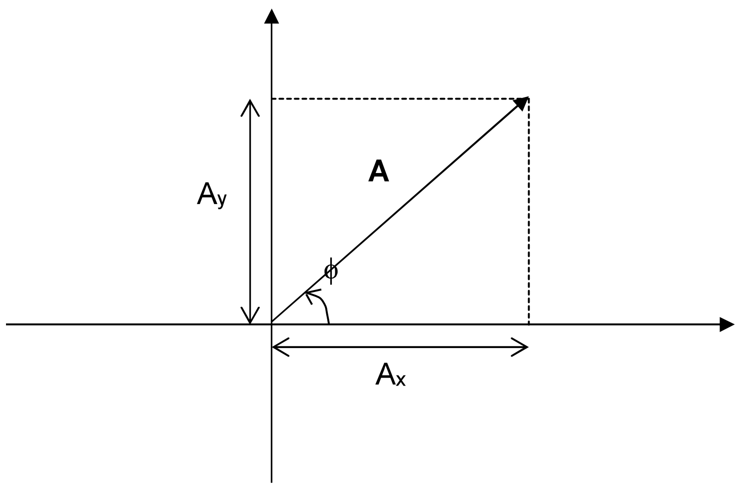
2.5.1. The concept of vector; vector representations

A vector is an entity with a size and a direction. In DiaMech we restrict ourselves to vectors in a plane, i.e. vectors in two dimensions.

There are two typographical conventions for representing vectors. In this chapter, vectors are represented as Bold Capitals or letters. The other convention, also used in other parts of DiaMech, is an italic lower-case letter or Capital with superimposed arrow.

In A figure, there are also two representations: an arrow in a Cartesian coordinate system from the origin to (X, Y), the coordinates of the point of the arrow, or the two components of the vector along the X- and Y- axes. A_X and A_Y.

The length of the arrow, A, and the angle φ with the X-axis, give both size and direction of the vector.



$$\varphi = arctg\frac{A_y}{A_x}; \qquad A_x = |\boldsymbol{A}| \cos\varphi; \qquad A_y = |\boldsymbol{A}| \sin\varphi$$

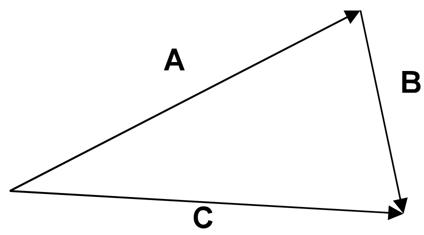
A unit vector has length 1 and is used to indicate a direction, often the direction of the one of the coordinate axes e_X or e_Y.

With the aid of the unit vectors in the directions of each of the axes, we can also write the relation between the vector **A** and its components as follows:

$\mathbf{A} = A\,\mathbf{e}_x + A_y\,\mathbf{e}_y$

This means that **A** is considered as the sum of two vectors lying along the coordinate axes and having the length of the components of **A**.

2.5.2. Adding and subtracting vectors



Denoting vectors by bold fonts, we define the addition of vectors: **A + B = C** or **B = C – A.** Using the convention that **–A** indicates a vector of the same length as **A** but of opposite direction, we can also write **B = (-A) + C = C – A**

The components of a vector are defined in a Cartesian coordinate system:

Each of the components A_x and A_y can be either positive or negative. The relations between the components and the vector and its direction are as follows:

With the aid of the unit vectors in the directions of each of the axes, we can also write the relation between the vector **A** and its components as follows:

A = A **e x** + A_y **e y**

This means that **A** is considered as the sum of two vectors lying along the coordinate axes and having the length of the components of **A**.

For **circular motion** it is practical to use Polar coordinates, the vectors **r** and **t, but these are not included in DiaMech**.

2.5.2.1**.** Properties of vectors

2.5.2.1.A.

Test Statements	T/F	Because
1. In some cases it is possible to add three vectors of non-zero length to give zero	**T**	1. When the three vectors, pictured as arrows of given length and direction, can be joined up head to tail to form a triangle, their sum is zero.
2. If any number of vectors of fixed length are given, one can always choose the directions of these vectors in such a way that their sum is zero.	**F**	2. The zero sum is only possible if no vector is longer than the sum of the lengths of all the other vectors
3. Vector **A** is horizontal and vector **B** has any direction. The vertical component of the sum **A+B** is the same as the vertical component of the difference **B-A**	**T**	3. Vector **A** has no vertical component, so the vertical component of both **A+B** and **B-A** equals the length of **B**.

2.5.2.1.B

Test Statements	T/F	Because
1. If the sum of two vectors equals zero, then the two vectors have the same length.	**T**	1. The two vectors are of equal length and opposite direction
2. The two vectors **A** and **B** make an angle ϕ with each other. Their sum, **C = A + B**, has a maximum when $\phi = 90^o$	**F**	2. **C** has a maximum length when $\phi = 0$.
3. If the angle between the two vectors **A** and **B** is varied while the lengths of both vectors are kept constant, then the direction of the sum vector **C = A + B** varies but the length of **C** remains constant.	**F**	3. The length of **C** depends on the angle between the vectors **A** and **B** as well as on their lengths

2.5.2.1.C

Test Statements	T/F	Because
1.If two vectors **A** and **B** are given, it is always possible to find a third vector **C** such that **A + B + C =** 0	**T**	1. **C = -(A + B)**
2. Vector **A** is horizontal and vector **B** vertical, the vertical component of the sum **A+B** is the same as the vertical component of the difference **A - B**	**F**	2. **A** has no vertical component, so the vertical component of the sum is B, and the vertical component of the difference **A – B** is –B.

2.5.3. Vector multiplication; scalar product

The **scalar product** of two vectors **A** and **B** situated in the *xy* – plane is $(\boldsymbol{A} \cdot \boldsymbol{B}) = A_x B_x + A_y B_y$

It is also possible to express the scalar product of **A** and **B** in the lengths of the vectors: $(\boldsymbol{A} \cdot \boldsymbol{B}) = AB \cos \phi$, where φ is the angle between the directions of **A** and **B**.

2.5.3.1, Properties of the scalar product

2.5.3.1.A. Vector **A** has length *a* and makes an angle of 60° with the positive *x*-axis of a given coordinate system. Vector **B** has a length *b* and makes and angle of 210° with the positive *x*-axis. And *a = 3b.*

The following statements can be made about the scalar product $(\mathbf{A} \cdot \mathbf{B})$:

Test Statement	T/F	Because
1. $\boldsymbol{A} \cdot \boldsymbol{B} = -\boldsymbol{B} \cdot \boldsymbol{A}$	**F**	1. The scalar product does not change if the order of the vectors is changed.
2. $\boldsymbol{A} \cdot \boldsymbol{B} = -\frac{3\sqrt{3}}{2}\ b^2$	**T**	2. Indeed. *cos150°* = -*cos30°* = -$\sqrt{3}/2$

2.5.3.1.B. Vector **A** has length *a* and makes an angle of 30° with the positive *x* axis of a given coordinate system. Vector **B** has a length *b* and makes and angle of 240° with the positive *x* axis. *a:b = 2:3.*

The following statements can be made about the scalar product $(\mathbf{A} \cdot \mathbf{B})$:

Test Statement	T/F	Because
1. $\boldsymbol{A} \cdot \boldsymbol{B} > 0$	**F**	1. The angle between **A** and **B** is larger than 90°, so cosφ < 0 and $\mathbf{A} \cdot \mathbf{B} < 0$
2. $\boldsymbol{A} \cdot \boldsymbol{B} = -\frac{1}{3}\ b^2\sqrt{3}$	**T**	2. The scalar product is AB *cos 150 °* = - AB *cos 30 °*

2.5.3.1.C. Vector **A** has length *a* and makes an angle of 45° with the positive *x* axis of a given coordinate system. Vector **B** has a length *b* and makes and angle of 270° with the positive *x* axis. *a:b = 2:1.*

The following statements can be made about the scalar product $(\mathbf{A} \cdot \mathbf{B})$:

Test Statement	T/F	Because
1. $\boldsymbol{A} \cdot \boldsymbol{B} = \boldsymbol{B} \cdot \boldsymbol{A}$	**T**	1. The scalar product does not depend on the order of the two vectors in the product.
2. $\boldsymbol{A} \cdot \boldsymbol{B} = -2\ b^2$	**F**	2.. The scalar product is AB *cos 135 °* = - AB *cos 45°* = - $b^2 \sqrt{2}$

2.5.3.1.D. Vector **A** has length *a* and makes an angle of 180º with the positive *x* axis of a given coordinate system. Vector **B** has a length *b* and makes and angle of 30º with the positive *x* axis. *a:b = 2:3.*

The following statements can be made about the scalar product $(\mathbf{A} \cdot \mathbf{B})$:

Test Statement	T/F	Because
1. $\boldsymbol{A} \cdot (-\boldsymbol{B}) = -(\boldsymbol{B} \cdot \boldsymbol{A})$	**T**	1. Changing the direction of one of the vectors changes the angle φ between the vectors by 180º, and (cos φ) changes sign
2. $\boldsymbol{A} \cdot \boldsymbol{B} = \frac{1}{3} b^2 \sqrt{3}$	**F**	2. The scalar product is AB *cos 150 º* = - AB *cos 30º* = - *(1/3) b² √3*

2.5.3.1.E. Vector **A** has length *a* and makes an angle of 120º with the positive *x* axis of a given coordinate system. Vector **B** has a length *b* and makes and angle of 60º with the positive *x* axis. *a:b = 1:2.*

The following statements can be made about the scalar product $(\mathbf{A} \cdot \mathbf{B})$:

Test Statement	T/F	Because
1. $\mathbf{A} \cdot \mathbf{B} \leq \mathbf{B} \cdot \mathbf{A}$	**F**	1. The scalar product is independent of the order of the vectors
2. $\mathbf{A} \cdot \mathbf{B} = a^2$	**T**	2. The scalar product is AB *cos 60 º* = *a² = b²/4*

DiaMech Chapter 3 Kinematics

This is Chapter 3 of DiaMech, a test of Knowledge and Skills in Elementary Mechanics. Please read the introductory Chapter 1 before using this chapter.

3.1. The concepts of position, velocity, and acceleration

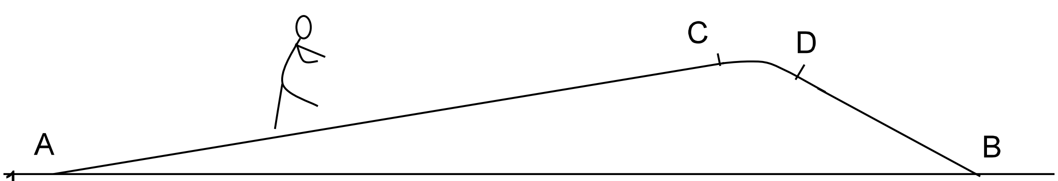
Kinematics is the description of the motion of **objects** in a chosen coordinate system. **In this chapter** we restrict ourselves to motion of a (point) object in two dimensions; this means motion in a plane. The variables we use are **the position**, **the velocity** and **the acceleration.** These variables are **vectors** in the plane of the motion. The **speed** is the magnitude of the velocity, and thus a **scalar**.

Knowledge and skills tested in this chapter:

Knowledge	Skills
• The concepts of position, velocity and acceleration are vectors	• Ability to describe vectors in a cartesian coordinate system. • Ability to interpret graphs of speed and acceleration versus time.

3.1.1.

A girl is running up a hill and down on the other side, from A to B**.** A and B are at the same level; the hill is not symmetric but the slopes are straight lines; only the top is rounded. Her steps are all of the same length, *l* m; she keeps up a constant rate of *n* steps per minute; it takes her *t* minutes to run from A to B.



3.1

Test Statement	T/F	Because:
1. The girl is running with a constant speed	**T**	1. The number of steps per seond and the length of the steps are constant, thus so is her speed
2. The velocity of the girl is constant all the way from A to B	**F**	2. The velocity changes direction between C and D, therefore it is not constant all the time
3.The vertical component of the velocity is numerically greater when the girl is running downhill than when she is running uphill	**T**	3. The magnitude of the velocity is constant, but downhill is steeper than uphill, therefore the vertical

		component is numerically greater in that part.
4. The girl has no acceleration while she is running uphill	**T**	4. Both magnitude and direction of her velocity are constant, therefore there is no acceleration
5. While the girl is passing over the top of the hill, from C to D, her average acceleration is vertically downwards	**F**	5. Her average acceleration has both a horizontal and a vertical component

3.1.1.B

Test Statement	T/F	Because:
1. The speed of the girl is greater downhill than uphill	F	1. The number of steps per minute and the length of the steps are the same as when she is running uphill
2. The velocity of the girl is constant all the way from A to C.	T	2. Both magnitude and direction of her velocity are constant
3. The vertical component of the velocity takes on both positive and negative values while the girl runs from A to B	T	3. he vertical component of the velocity is positive while she is running uphill and negative while she is running downhill
4. Running downhill from D to B, the girl has an acceleration in the downwards direction	F	4. From D to B her velocity is constant in magnitude and direction; she has no acceleration.
5. When the girl stops at B, she experiences a horizontal acceleration	F	5. Her acceleration has both a vertical and a horizontal component

3.1.1.C

Test Statement	T/F	Because:
1. The girl is speeding up when running downhill	**F**	1. The number of steps per minute and the length of the steps remain constant
2. The velocity of the girl changes while she is passing the top of the hill from C to D.	**T**	2. The velocity changes direction while she is passing the top of the hill
3. The velocity has a constant horizontal component all the way from A to B	**F**	3. Downhill is steeper therefore the horizontal component is smaller.
4. The motion of the girl from A to B is not accelerated	**F**	4. The motion is accelerated while the girl passes over the hill
5. While the girl is passing over the top of the hill, her acceleration has a -component downwards	**T**	5. The velocity does change downwards

3.1.1.D

Test Statement	T/F	Because:
1. The total change of position of the girl is a vector with a magnitude smaller than *nlt*	T	1. Her total change of position is the vector from A to B, which has a smaller magnitude than *nlt*
2. The velocity has a constant horizontal component all the way from A to B	**F**	2. The magnitude of the velocity is constant, but downhill is steeper, therefore there the horizontal component is smaller
3. The numerical value of the vertical component of the velocity increases while the girl is running downhill	**F**	3. Both components of the velocity remain constant while the girl is running downhill
4. Running uphill the girl has an acceleration upwards	**F**	4. Both magnitude and direction of the velocity are constant, therefore there is no acceleration
5. While the girl is passing over the top of the hill, from C to D, the horizontal component of het velocity remains the same	**F**	5. The horizontal component of her velocity first increases on the top, and from D onward is lower than during the uphill, as the downhill slope is steeper.

3.1.1.E.

Test Statement	T/F	Because:
1. The total change of position of the girl is vector with magnitude *nlt*.	**F**	1. The total distance run is *nlt* but the girl moves from A to B, which is a smaller than *nlt*.
2. The vertical component of the velocity takes on both positive and negative values	**T**	2. Uphill the velocity has a upwards component, and downhill a downwards component
3. The horizontal component of the velocity is numerically smaller when the girl is running downhill than when she is running uphill	**T**	3. Downhill is steeper than uphill, and the magnitude of the velocity is constant, therefore the horizontal component is smaller running downhill
4. Running downhill the girl has no acceleration	**T**	4. Both magnitude and direction of the velocity are constant therefore she has no acceleration
5. While the girl is running over the top of the hill, from C to D, her motion is accelerated.	**T**	5. While the girl is running over the top, the direction of the velocity changes, therefore there is an acceleration

=

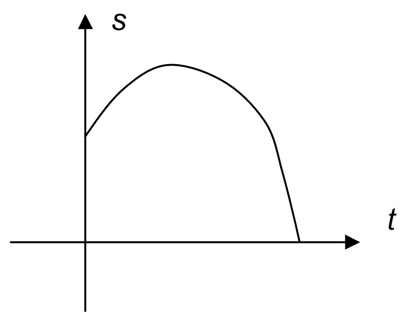
3.2. Interpretation of graphs about motion

This sub-chapter uses a different type of test questions. We first give a graph showing either position, velocity or acceleration versus time. In all these graphs, the time is shown along the horizontal axis. We then present a number of real-world **Test Situations**, and for each we ask **the True-False question: can this graph apply to that test statement?**

3.2.1. Graphs of Position versus time

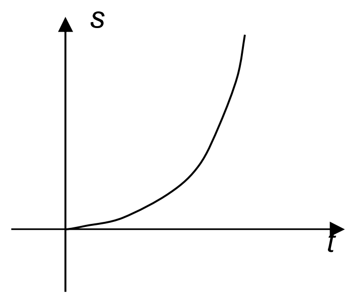
In these graphs,, the time is shown along the horizontal axis, and **the position is shown as a distance *s* along the vertical axis of the coordinate system.** That coordinate axis may represent any direction in the real world; When that real-world axis is vertical, the origin *s=0* is set at ground level. Curves are sketched, and may not follow the exact mathematical shape.

3.2.1.A. This is a graph of distance s against time t. Below are four test situations with a statement for each of them. Can this graph represent the motion described in the Test Statement (True or False)?



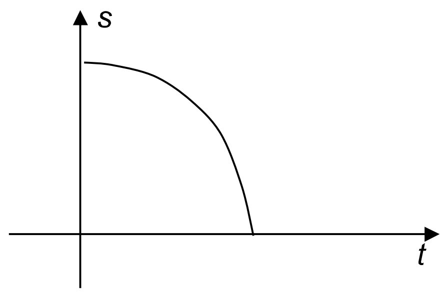
Test Statement	T/F	Because:
1. The vertical position of a ball thrown vertically upwards from the edge of a roof and falling to the ground	**T**	1. Describes the height of the ball correctly
2. The movement of a ball thrown vertically upwards from the edge of a roof and falling onto a ledge above the starting position	**F**	2. The graph shows it falls lower.
3. The vertical position of a ball thrown upwards at an elevation of 45^o from a roof and falling to the ground	**T**	3. Describes the height of the ball correctly
4. The trajectory of a ball thrown upwards at an elevation of 45^o from a roof and falling to the ground	**F**	4. The graph is not a trajectory; the horizontal coordinate is time. not position.

3.2.1.B. This is the graph of distance *s* against time *t*. Below are four test situations with a statement for each of them. Can this graph represent the motion described in the Test Statement (True of False)?



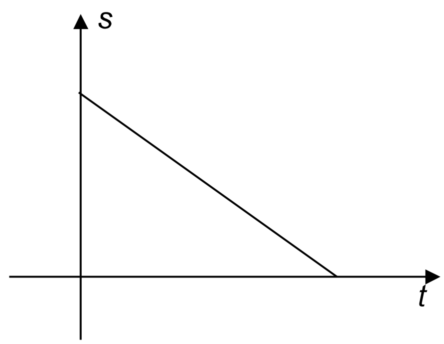
Test Statement	T/F	Because:
1. The position of a car passing s = 0. at t = 0. and moving with constant speed.	**F**	1. At constant speed ***s*** would be proportional to ***t***
2. The position of a car moving with constant acceleration	**T**	2. The slope, which shows the speed, increases with time
3. The position of a stone thrown vertically upwards from ground at t = 0	**F**	3. The l slope, which shows the speed, decreases to zero
4. The path followed by a car accelerating through a curve	**F**	4. The coordinate ***t*** is time, it does not show a position

3.2.1.C **. This is a graph of distance s against time t. Below are four test situations with a statement for each of them. Can this graph represent the motion described in the Test Statement (True or False) ?**



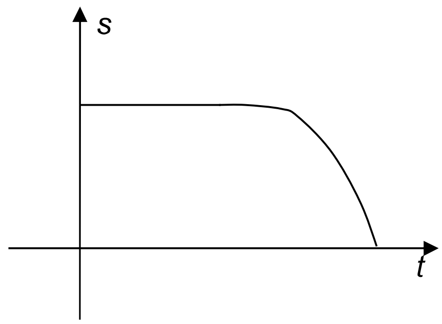
Test Statement	**T/F**	**Because:**
1. The position of a stone falling vertically from a roof to the ground	T	1. The increasing slope shows the increasing speed.
2. The trajectory of a stone thrown horizontally from a roof and falling to the ground	F	2. The horizontal coordinate ***t*** is time, not position
3. The height of the top of a tree falling over	T	3. The downwards speed increases during the fall.
4. The position of a car moving with constant acceleration in the direction of decreasing values of *s*	T	4. The speed in the direction of -*s* constantly increases .

3.2.1.D **. This is a graph of distance s against time t. Below are four test situations with a statement for each of them. Can this graph represent the motion described in the Test Statement (True or False) ?**



Test Statement	**T/F**	**Because:**
1. The position of a stone falling vertically from a roof to the ground	F	1. The slope (showing the speed) is constant
2. The position of a weight lowered on a rope with constant speed from a roof to the ground	T	2. The slope (showing the speed) is negative and constant.
3. The path of a stone falling from an inclined roof to the ground	F	3. The horizontal coordinate is time; the graph cannot show a path in space
4. The height of a parachutist in the last part of her jump	T	4. In this part of the jump her speed of fall is constant.

3.2.1.E **. This is a graph of distance s against time t. Below are four test situations with a statement for each of them. Can this graph represent the motion described in the Test Statement (True or False) ?**

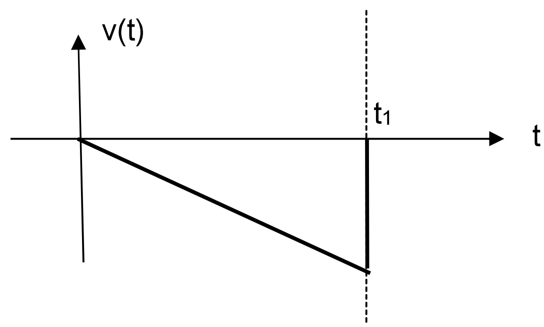


Test Statement	T/F	Because:
1. The position of a pail hanging at a rope at a constant height above the ground, and then being lowered to the ground at a constant speed	F	1. The speed in the diagram is not constant but increasing in the negative direction
2. The path of a car driving with constant speed along a straight road and then around a bend	F	2. The graph shows a position that is first stationary,
3. The position of a pail hanging at a rope at a constant height above the ground, until the rope breaks at a certain moment, and the pail falls to the ground.	T	3. Position well represented both before and after the rope breaks
4. The trajectory of a girl running with constant speed and then jumping over the edge of a rock	F	4. The graph shows a position that is first stationary,

3.2.2. Graphs of Velocity versus Time

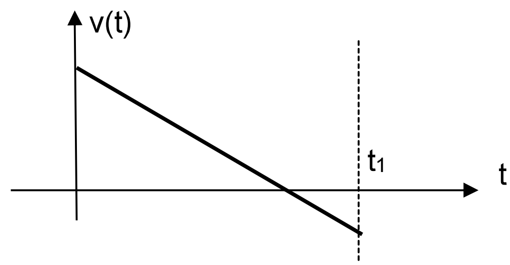
In all these graphs, the time is shown along the horizontal axis. The test situations are so chosen that we only need one of the components of the velocity vector (vertical, horizontal) or the length of the vector (i.e. the speed) **That component of the velocity is shown as a distance *v* along the vertical coordinate axis.** That velocity component may have any direction in the real world. The origin is set at velocity zero. . Curves are sketched, and may not follow the exact mathematical shape.

3.2.2.A. **This is a graph of vertical velocity *v (upwards)* against time t. Can this graph represent the motion described in the Test Statement (True or False) ?**



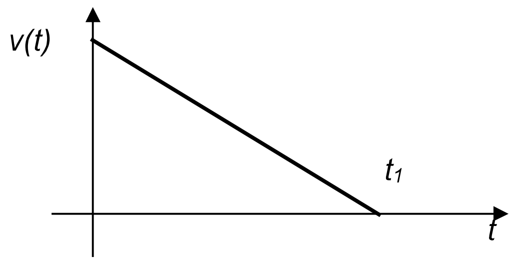
Test Statement	T/F	Because:
1. A stone falling from a roof to the ground	T	1. Indeed: increasing negative velocity and sudden stop
2. A ball thrown downwards from a roof to the ground	F	2. The graph shows a velocity starting from zero
3. A ball rolling down an irregular slope, and stopped at the end,	F	3. The graph shows a uniformly increasing velocity, that does not happen on an irregular slope
4. A submarine diving to the bottom of the sea	F	4. No captain would allow the submarine to hit the bottom at high speed
5. A pail lowered with a constant velocity into a well	F	5. The graph shows an increasing negative velocity

3.2.2.B . **This is a graph of vertical velocity *v (upwards)* against time t. Can this graph represent the motion described in the Test Statement (True or False) ?**



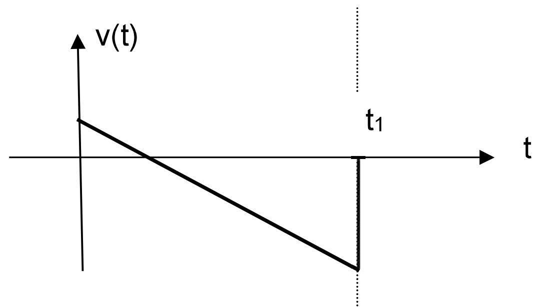
Test Statement	T/F	Because:
1. Part of the movement of a rocket after the motor has just stopped	T	1. The rocket, still moving upwards, is now in free fall.
2. A ball thrown upwards from the ground and then falling onto a roof at $t = t_1$	T	2. The ball moved up longer than down, so it ended higher than is started
3. A stone falling from a certain height into a well, hitting the water surface at $t = t_1$	F	3. The stone never moved upwards
4. A pail lowered with a constant velocity into a well hitting the water surface at $t = t$	F	4 The pail never moved upwards and its velocity is constant
5. An object is lying on an incline without friction and is pushed upwards, and then slides down	T	5 This object moves first upwards and then down

3.2.2.C**. This is a graph of vertical velocity *v (upwards)* against time t. Can this graph represent the motion described in the Test Statement (True or False) ?**



Test Statement	T/F	Because:
1. A ball thrown upwards from a roof and then hitting the ground at $t = t_1$	F	All motion on the graph is upwards
2. A ball thrown upwards from the ground and then hitting a roof at $t = t_1$	T	The ball stops at its highest point
3. A ball rolling down an incline, reaching the bottom at $t = t_j$,	F	All motion on the graph is upwards
4. A stone falling from a certain height above ground into a well	F	All motion on the graph is upwards
5. A car driving up a hill, slowing to a stop	T	The speed upwards decreases regularly to standstill

3.2.2.D **This is a graph of vertical velocity *v (upwards)* against time *t*. Can this graph represent the motion described in the Test Statement (True or False) ?**

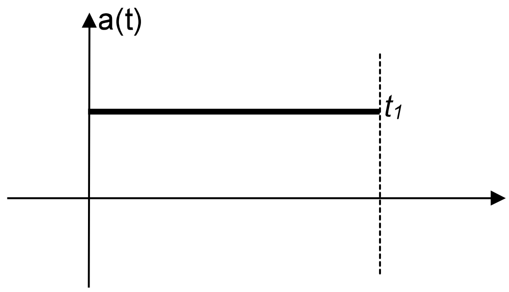


Test Statement	**T/F**	**Because:**
1. A stone falling from the wall of an empty well, reaching the bottom at $t = t_1$	F	Stone starts at v =f0
2. A ball thrown vertically upwards from a roof then falling and reaching the ground at $t = t_1$	T	Ball falls longer than it rises, therefore must land lower
3. A ball thrown vertically upwards from the ground, then falling and hitting ground again at $t = t_1$	F	Ball falls longer than it rises, so must land lower
4. A ball thrown vertically upwards from a position close to the edge of an empty well, then falling into the well and hitting the bottom surface at $t = t_1$	T	Ball Falls longer than it rises, therefore lands lower
5. A child going off a water slide into a swimming pool, hitting the surface of the pool at $t = t_1$	F	Child does not start upwards

3.2.3. Graphs of Acceleration versus Time

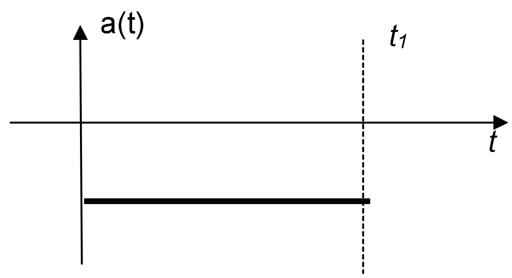
In all these graphs, the time is shown along the horizontal axis. The test situations are so chosen that we only need one of the components of the acceleration vector (vertical, horizontal or the length of the vector). **That component of the acceleration is shown as a distance *a* along the vertical axis of the coordinate system, *a=0* being the origin.** That coordinate axis may represent any direction of acceleration in the real world. Curves are sketched, and may not follow the exact mathematical shape .

3.2.3.A. This is a graph of acceleration *a (upwards or forwards is positive)* against time t. Can this graph represent the motion described in the Test Statement (True or False) ?



Test Statement	T/F	Because:
1. A ball falling from the level of a roof and reaching the ground at $t = t_1$	F	The ball never accelerates upwards
2. A ball thrown upwards from the ground, then falling back to ground	F	The ball never accelerates upwards
3. A car constantly increasing its velocity from zero to some final velocity	T	We say "The car accelerates".
4. A rocket constantly increasing its velocity from a certain velocity v_0. at $t = 0$. to some final velocity	T	"The rocket accelerates"
5. A car decreasing its velocity from a certain velocity at $t = 0$. to zero	F	The acceleration is never negative.

3.2.3.B. This is a graph of acceleration *a (upwards or forwards is positive)* against time t. Can this graph represent the motion described in the Test Statement (True or False) ?

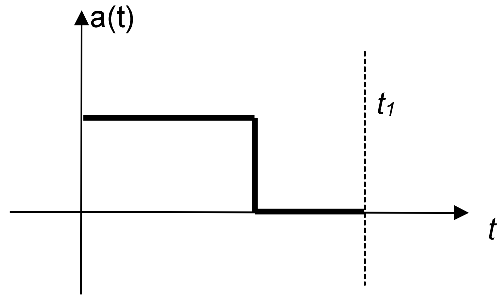


Test Statement	T/F	Because:
1. A ball falling from the level of a roof to the ground	T	Constant downwards acceleration
2. A ball thrown upwards from the ground, then falling back to ground	T	Constant downwards acceleration
3. A car increasing its velocity from zero to some final velocity	F	Graph shows no positive acceleration
4. A car braking to stop for a railway crossing	T	Constant acceleration, slowing down
5. A block sliding on a horizontal surface with some friction.	**T**	Constant acceleration, slowing down

3.2.3.C. This is a graph of acceleration *a (upwards or forwards is positive)* against time t. Can this graph represent the motion described in the Test Statement (True or False) ?

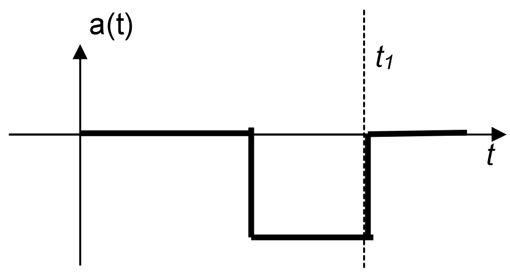
Test Statement	T/F	Because:
1. A stone falling from the edge of a roof to the ground	F	The graph shows zero acceleration
2. A ball thrown upwards from the ground	F	The graph shows zero acceleration
3. A car driving with constant velocity	T	The graph shows zero acceleration
4. A car braking to stop for a railway crossing	F	The graph shows zero acceleration
5. A ball rolling over a horizontal surface with no rolling resistance	T	The graph shows zero acceleration

3.2.3.D. This graph plots acceleration *a (upwards or forwards is positive)* against time t. Can this graph represent the motion described in the Test Statement (True or False) ?



Test Statement	T/F	Because:
1. A ball thrown vertically upwards to a certain height and then falling freely	F	The graph shows no downwards acceleration
2. A car accelerating from rest to a certain speed, then driving on with constant velocity.	T	Indeed: acceleration positive ,, then zero
3. A stone falling from a roof to the ground, then resting on the ground	F	The graph shows no downwards acceleration
4. A car driving with constant velocity, then braking for a railway crossing	F	The graph shows no negative acceleration
5. A rocket increasing its velocity from $v_{0.}$ to $v_{1.}$, then travelling with constant velocity	T	Indeed: acceleration positive ,, then zero

3.2.3.E. This is a graph of acceleration *a (upwards or forwards is positive)* against time t. Can this graph represent the motion described in the Test Statement (True or False) ?



Test Statement	T/F	Because:
1. A train running with constant velocity, then braking to stop at a station	T	Indeed: Graph shows first constant velocity, then braking,
2. A train running with constant velocity, braking, and continuing more slowly	T	Indeed, the graph says nothing about stopping
3. A car waiting for a traffic light, then accelerating	F	The graph never shows positive acceleration
4. A ball thrown vertically upwards to a certain height and then falling freely	F	The graph shows acceleration first zero, then downwards
5. A piece of wood falling into a well, then floating on the water	F	The graph shows no upwards acceleration when the wood hits the water

DiaMech Chapter 4

Dr. Monica Ferguson-Hessler TUE (retired)

Introduction

This is Chapter 4 of DiaMech, a test of Knowledge and Skills in Elementary Mechanics. Please read the introductory Chapter 1 to DiaMech before using this chapter.

Knowledge and skills tested

Knowledge	Skills
- Concepts of systems, surroundings, interaction - Concept of an inertial system of coordinates. - Concept of force as an interaction. - Concept of linear momentum relevant interaction, conservation of linear momentum - Newton's First Law - Newton's Second Law - Newton's Third Law	- Defining an inertial coordinate system suitable for the solution of the given problem. - Identifying all forces acting on and within a system of bodies. - Distinguishing internal forces in a system of bodies from external forces on the system. - Distinguishing forces on a system of bodies from forces by system on the surroundings. - Applying the concept of system to a dynamic situation - Analysing the influence of forces on the linear momentum of a system.

Content, general.

This chapter starts with a discussion of the concepts involved. First comes the notion of an inertial system of coordinates, as Newton's laws can only be understood easily if these are used/ Then we discuss Newton's three laws, force, mass, linear momentum, and acceleration.

Questions cover the skill of distinguishing inertial coordinate system from non-inertial coordinate systems, internal and external forces in/on a system, and the effect of these forces on the motion of the system.

Frictional forces and their influence are treated in separate paragraphs in chapter 5.

Because of the frequent misconceptions in this area of Mechanics, an appendix has been added, which treats three well-known misconceptions: "No forces act on a body at rest", "Force produces constant velocity", and "Heavy bodies fall faster".

4. The motion of objects

In this chapter, we are looking at the fundamental processes of mechanical interaction: how physical objects move when they exert forces on each other.

4.1. Inertial Coordinate Systems

In mechanics we are looking at objects that move around in space, and so each have a **position in space which changes with time**. We can use any system of coordinates to describe their positions, but only if we use an **inertial system of coordinates** do Newton's simple laws apply. That is any coordinate system that is not rotating or accelerated with respect to distant objects. That is not one coordinate system. If we have an inertial coordinate system; any coordinate system that has a constant velocity in any direction with respect to it is also inertial.

We often use a system connected to earth (or moving with constant velocity relative to the earth) as close approximation to an inertial system, but if we want to do celestial mechanics we use a coordinate system in which distant stars are stationary. The reason is that when described in accelerated or rotating coordinate systems objects experience virtual forces, not caused by interaction with the surroundings, think of the force pressing you forward or sideways in a braking or turning car! This does not mean that Newton's laws are any less valid if you use a non-inertial coordinate system, but only that their mathematical formulation becomes excessively complicated.

4.1.1. Test Units A - E Distinguishing inertial systems from non-inertial systems

4.1.1.A. **The following coordinate systems are inertial systems**

Test Statement	T/F	Because
1. A system fixed to the top of a table of the classroom	T	1. Yes, this is an inertial system
2. A system fixed to a car travelling with increasing speed along a motor way	F	2. No, this system has an acceleration relative to earth and is not inertial
3. A system fixed to a stone swung on the end of a string in a horizontal circle.	F	3. No, this system does not move with constant velocity relative to earth and is not inertial
4. A system fixed to a car breaking to stop for red light	F	4. No, this system has an acceleration relative to earth and is not inertial

4.1.1.B. **The following coordinate systems are inertial systems**

Test Statement	**T/F**	**Because**
1. A system drawn on the blackboard of the classroom	T	1. Yes, this is an inertial system.
2. A system fixed to a block moving freely down an inclined plane without friction	F	2. No, gravity gives this system an acceleration and it is not an inertial system
3. A system fixed to a lorry travelling with constant speed along a motor way	F	3. No, this is an inertial system only as far as there are no curves
4. A system fixed to a car driving round a sharp curve	F	4. No, this is not an inertial system' the axes change direction.

4.1.1.C. The following coordinate systems are inertial systems

Test Statement	T/F	Because
1. A system fixed to the top of a high building	T	1. Yes. his is an inertial system
2. A system fixed to a body which is falling freely from a very high building	F	2. No, the body has an acceleration relative to the ground, so this not an inertial system
3. A system fixed to a "merry-go-round"	F	3. No, this system is rotating, and is not an inertial system
4. A system fixed to the engine of a train moving with constant speed through a long straight tunnel	T	4. this is an inertial system (as long as the engine is not jolting).

4.1.1.D. **The following coordinate systems are inertial systems**

Test Statement	**T/F**	**Because**
1. A system fixed to a car driving down a long incline without braking	F	1. No, this system is accelerated and is not an inertial system
2. A system fixed to an elevator which is moving downwards with constant velocity	T	2. Yes, this is an inertial system.
3. A system fixed to an aeroplane which is landing on the runway	F	3. No, the aeroplane is braking so this system is accelerated and is not an inertial system

4.1.1.E. The following coordinate systems are inertial systems

Test Statement	T/F	**Because**
1. A system fixed to an elevator which is just starting to move upwards	F	1. The elevator is accelerating.
2. A system fixed to a car driving straight down a long incline, keeping the speed constant by braking on the motor	T	2. Yes, this is an inertial system
3. A system fixed to the helmet of a parachutist who has just jumped off his plane .	F	3. The parachutist is accelerating

4.2 Newton's Laws

First we have the objects. They have a mass, often called *m*, which can have any value, from immense to minute, and any density, from the densest metal to the most tenuous gas. In many practical problems it is not necessary to look at their physical size, and we approximate them by considering them as a "Point Mass."

Newton's first crucial insight was that an object that is alone in space far from other objects has no reason to do anything else than what it was already doing: moving in a straight line. That is Newton's first law

Then the objects can exert **forces on each other:** they may be in direct contact and pull or push each other, but they can also be some distance apart, and exert gravitational, magnetic or electrostatic forces on each other. Then they both deviate from the straight path at constant velocity of the lone object.

Around the end of the 17^{th} century Newton formulated three laws which have since become the core of understanding mechanical interactions. They are based on experiment and observation.

Newton's First Law: An object that is not subject to interactions with other objects continues its motion (velocity vector: speed and direction) indefinitely.

Newton's Second Law: An object of mass m that is subject to a force $\boldsymbol{F}$ (a vector) experiences an acceleration of $\boldsymbol{a}$ (a vector) given by $\boldsymbol{F} = m\,\boldsymbol{a}$. The force $\boldsymbol{F}$ is the sum of all the forces exerted on the object by other objects in the world. Newton' s first law is thus a special case of the second law,

Newton's Third Law: If an object A exerts a force $\boldsymbol{F}_{A\to B}$ on object B, then the object B exerts an equal and opposite force on object A, i.e. $\boldsymbol{F}_{A\to B} = -\boldsymbol{F}_{B\to A}$. That means that forces always come in pairs that are equal and opposite. The force between two bodies may act at a distance (e.g. gravity or an electric or magnetic field) or act only at a point of contact with the body (e.g. a spring or a support).

4.3 The notion of System

The word **system** is used in many areas of knowledge for indicating many different types of interaction.

In mechanics, **"system"** is used to indicate a well-defined part of the universe, specified by the collection of the objects you want to study at a particular moment. It is an instrument that you use to clarify your understanding of the physical situation. You select one part of the objects in the world to form the **system,** the rest of the world is outside the system, and is usually called **the surroundings**. Strictly speaking, Newton's laws may apply only to objects at the smallest scale, as atoms, but they can still be included in the system.

We can now divide all the forces that are acting between objects in the world into three groups.

- Forces acting between objects of the system, also called the **internal forces** in the system, Those forces are very real, but they have no influence on the motion of the system: they sum to zero, as both objects involved are within the system, and the forces cancel out (Newton's third law)
- Forces acting between objects in the "rest of the world" which do not affect our system
- Forces acting between an object of the system and an object in the surroundings, also called the **external forces on the system.**

When we are considering the interaction of the **system** with its **surroundings** this classification of forces into three groups is very useful. We only have to consider the force ***F***, being the sum of all external forces acting on the system, and ***a*** the acceleration of it center of gravity. (In DiaMech we do not discuss rotation of complex objects).

Choosing the system

When we are faced with a problem in a mechanical device, we are entirely free to define any parts of the device as 'systems' for closer study: the system is chosen to fit the interest of the onlooker. In fact, you may find out that solving a complex real-world problem will require you to use different systems at different times.

Remember that when looking at a system consisting of multiple objects, it is important to use *only forces acting on the system*. One starts by looking at forces acting at a distance (gravity, electric and magnetic forces) and then checks all points of contact between the body and the surroundings. Remember to exclude forces between objects of the system itself.

4.4. Conservation of linear momentum.

Linear momentum is a concept that plays a central role in classical mechanics, and that also obeys a conservation law. Linear momentum $\vec{p}$ is a property of a moving body, a vector defined by

$$\vec{p} = m\vec{v}$$

where *m* is the mass and $\vec{v}$ the velocity of the body. We treat *m* as a point mass located at the center of gravity of the body.

For a system consisting of two or more bodies we define the total linear momentum:

$\vec{p}_{tot} = \sum_i \vec{p}_i = \sum_i m_i \ \vec{v}_i$

One difference with conservation of money or mass is that **linear momentum is a *vector***. That means that any relation we write down for the vector $\vec{p}$ contains three separate relations, one for each component of $\vec{p}$ along the axes of a cartesian coordinate system. (For more information on coordinate systems refer to chapter 2.)

When a set of external forces F_i *(i = 1 ...m)* acts on a system of one or more moving bodies m_k *(k = 1 ...s)* during a time Δt , the linear momentum of the system is changed.

$\sum_i \vec{F}_i \ \Delta t = \Delta \ \sum_i m_i \vec{v}_i = \Delta \vec{p}_{tot}$

Now we consider a single mass m, and find the limit as *Δt* decreases to zero, :

$$\sum_i \vec{F}_i = \lim_{\Delta t \to 0} \frac{\Delta(m\vec{v})}{\Delta t} = m\frac{d\vec{v}}{dt} = m\ \vec{a}$$

This is an alternate way of writing **Newton's second law** as it applies to systems. The vector formula summarizes the relations between the forces acting on the body and its acceleration, one relation for each of the three coordinate directions.

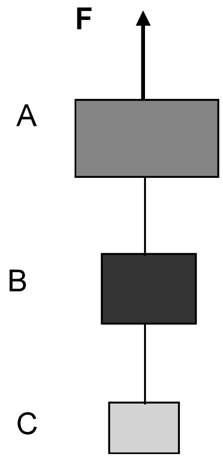
Note that the concept of *force always is an reciprocal interaction* between a body (or a system of bodies) and its surroundings. Forces always occur in pairs. Every force $\vec{F}_i$ **by the surroundings A on the body B** is always accompanied by an equal and opposite force -$\vec{F}_i$ **by the body B on the surroundings A, as expressed by** Newton's third law $\vec{F}_{A\to B} = -\vec{F}_{B\to A}$

When applying the Newton's second law to a chosen system it is important to **include only the forces acting on the system**, and not the forces by the system on the surroundings. In the discussion of the **law of conservation of linear momentum** it is important to realise that **force** is defined as an **interaction between two bodies** whereas **linear momentum *p*** = *m**v*** (vectors) is a **property of one body or system**. We see that this law of conservation is none other than Newton' s 2nd law ***F*** = *m* ***a***, where we express force as $\mathbf{F} = \frac{d\mathbf{p}}{dt}$. and the acceleration as $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2}$.

4.2.1.1. **The relation between forces and the acceleration of a system**

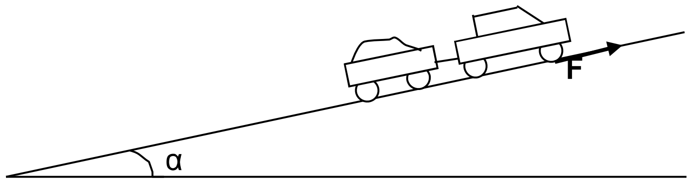
These are tested in the following Test Units A - E

4.2.1.1.A Three bodies, A, B, and C are connected by strings, A to B and B to C as shown in the figure. A constant force **F** acts upwards on the uppermost body A. The mass of A is *3m*, the mass of B is *2m*, and the mass of C is *m*.



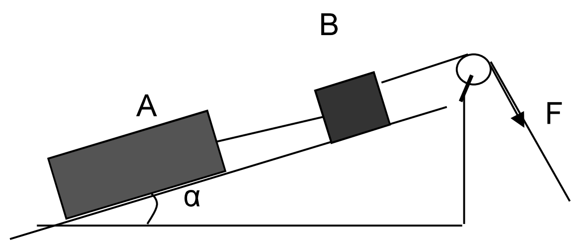
Test Statement	T/F	Because
1. The three bodies move upwards	F	1. No, that is not always true. If *F* is smaller than *6mg,* they all accelerate downwards.
2. A, B, and C move with constant acceleration	T	2. They may move upwards or downwards, but always with a constant acceleration
3. When the system is at rest the tension in string A-B is greater than the tension in string B-C	T	3. The net force of these two strings must balance the gravity acting on B
4. The tensions in the strings A-B and B-C are internal forces and therefore independent of the acceleration given to the system	F	4. In rest, the forces in the strings balance the gravity acting on each body. With acceleration, they change to give each block the net force needed for the acceleration
5. If **F** = 0, then A, B, and C move with constant velocity	F	5. If **F** = 0. then all three bodies fall freely with acceleration *g*.

4.2.1.1.B. A car is driving up a hill with inclination α, pulling a loaded trailer. The car has front wheel drive, and the total tangential force between the front wheels and the surface of the road is **F**. There is no rolling friction. The mass of the car is *m* and that of the loaded trailer *km*.



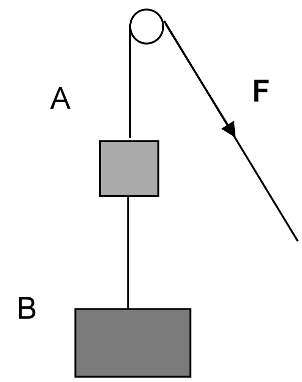
Test Statement	T/F	Because
1. As long as **F** remains constant, the car moves with constant velocity	F	1. If a **net force** acts on the car, it will be accelerated.
2. The magnitude of the force on the wheels required to give the car an acceleration *a* is *ma*	F	2. The total force on the car is *ma,* but there are two downward forces as well, a component of gravity and the pull of the trailer.
3. It is possible to calculate the acceleration of the car for a given **F** without first finding the force between the car and the trailer.	T	3. By choosing car + trailer as system, one can find the acceleration caused by the net force along the hill, $F-(k+1)mg\ sin\alpha$
4. The tension in the connection between the car and the trailer depends only on the mass *km.*	F	4. The tension also depends on the acceleration and therefore on **F**
5. If the road becomes very slippery and **F** becomes zero, then the car will slow down and then move downwards with constant velocity	F	5. The car will slow down and then move down the hill with increasing velocity, being accelerated by gravity.

4.2.1.1.C. Two blocks, A and B, move on a frictionless plane, inclined at an angle α (see figure). The mass of A is *3m* and that of B *m.* A and B are connected by a string, and another string runs from B over a frictionless pulley and is free at the other end. A person is pulling at the free end with a constant force F, and A and B are moving upwards. The mass and friction of the pulley is neglected



Test Statement	T/F	Because
1. Can A and B keep moving with constant velocity?	T	1. Only if F equals the forces that gravity exerts on A + B along the slope.
2. To maintain the motion we need $F \geq 4mg$	F	2. If $F \geq 4mg\ sin\alpha$ A and B will accelerate up the plane
3. The tension in the string between B and the pulley equals F	T	3. The pulley only serves to change the direction of the force
4. The acceleration of B is F/m	F	4. The net force acting on B is smaller than F
5. If the person let go of the string, then A and B come to rest and then slide down the incline with constant acceleration.	T	5. The component of gravity parallel to the plane will give A and B a constant acceleration downwards.

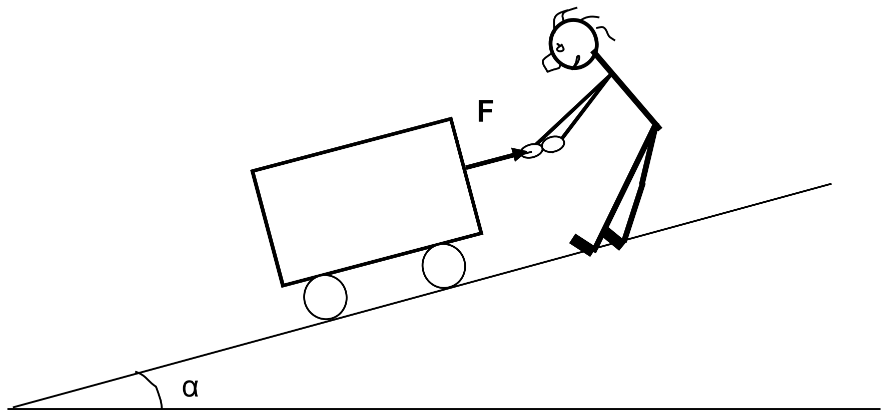
4.2.1.1.D. Two blocks, A and B, have masses *m* and *2m* respectively. They are joined by a string, and A is hanging by a 2nd string from a pulley. A person is pulling on this string with a constant force **F** (see figure)



Test Statement	T/F	Because
1. A and B move with constant acceleration	T	1. Yes, this is correct.
2. The tension in the string between A and the pulley depends on the angle that **F** makes with the vertical.	F	2. The pulley changes the direction of the force in the string, but not the magnitude
3. Can A and B move downwards if **F** is ≥ *3mg ?*	T	3. Yes, but the acceleration is upwards.
4 The tension in the string between A and B is the same as the tension in the string between A and the pulley	F	4. The difference between these tensions gives the force needed to accelerate A and support its weight.
5. The tension in the string between A and B is *2mg*	F	5. he tension in this string depends on the acceleration of B

4.2.1.1.E.

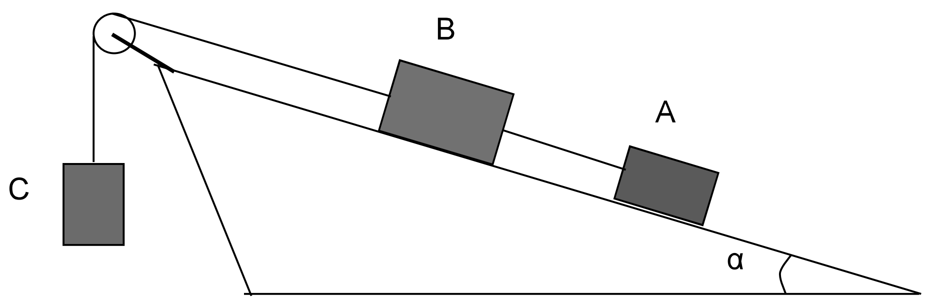
A man is guiding a loaded cart down a steep hill (see figure). The cart is rolling down too fast and the man is slowing it down. There is no friction in the wheels of the cart. The man is digging his heels in to be able to assert a force of magnitude F parallel to the surface of the hill. The slope of the hill is α. The mass of the man is *m* and that of the loaded cart *km* .



Test Statement	T/F	Because
1. Only if F is large enough will the man succeed in slowing down the motion of the cart.	T	1. If **F** is too small the cart will accelerate downwards.
2. In order to slow down the cart the man must exert a force on the ground parallel to the surface which is ≥ $(k+1)mg$	F	2. The force must be ≥ $(k+1)mg \sin \alpha$
3. The force on the rope needed to slow down the cart decreases when the velocity of the cart decreases	F	3. The force must exceed the component of the weight of the cart down the slope, and this is constant.
4. The shorter the distance available to slow down the cart, the greater the force F must be.	T	4. The greater the magnitude of the acceleration, the greater force is required.
5. If the man suddenly lets go of the rope, then the cart will continue to move down the slope with constant acceleration.	T	5. The acceleration downwards only depends on the angle of the slope, and is $g \sin \alpha$

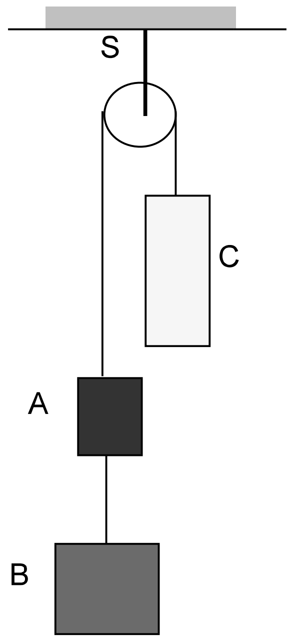
4.2.1.2 The motion of complex systems

4.2.1.2.A Two blocks, A and B, are lying on a frictionless inclined plane. The angle of the slope α; *sin α =0.2 .* A and B are connected by a string, and B is connected by another string to a third block C via a frictionless pulley. C is hanging freely on this string (see figure). The mass of A is 2*m*, the mass of B is 3*m*, and that of C is *2m*. The mass of the pulley is neglected.



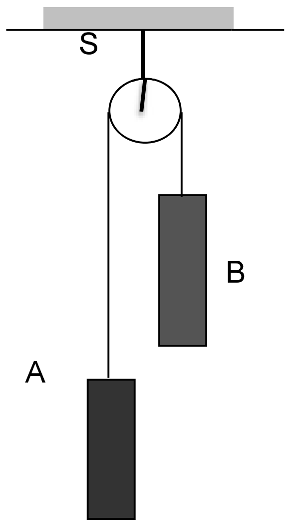
Test Statement	T/F	Because
1. If C is held at rest and then let free, A and B will move down the slope	F	1. The total component of gravity acting on A and B along the slope is smaller than *2mg*, so they will move up
2. If C is held at rest and then let free, A and B will move with constant acceleration.	T	2. There is a constant net force acting on A and B
3. The tension in the string between A and B equals the tension in the string between the pulley and B	F	3. The tension in the upper string is larger; it must accelerate B, overcome B’s friction, and also pull A
4. If α were to be increased to a value greater than 30 º, and C let free from rest, then A and B would move down the slope	T	4. In that case the component of gravity of A + B along the slope is greater than *2mg*
5. If the string that connects C to the rest of the system were to break, then A and B would move down the slope with constant velocity	F	5. ÍA and B would slide down the slope with constant acceleration.

4.2.1.2.B. The masses A, B, and C are connected with strings and hanging from a frictionless pulley as shown in the figure. The pulley hangs from the ceiling on a support S. The masses of A, B and C are m, $3m$, and M respectively. $M > 4m$. The mass of the pulley is small and is neglected



Test Statement	T/F	Because
1. If C is held at rest and then let free, A and B will move upwards with constant velocity.	F	1. A and B will move with constant acceleration.
2. The force required to support C at rest is $(M - 4m)g$	T	2. Indeed
3. If the system (A+B+C) is left free to move, then the tension in the string between A and the pulley equals the tension in the string between C and the pulley	T	3. The tension on both sides of the pulley is the same.
4. If the system (A+B+C) is left free to move, then the tension in the string between A and B equals the tension in the string between A and pulley	F	4. The tension between A and B is smaller than the tension between A and the pulley
5. If the system (A+B+C) is left free to move, then the tension in the support S holding the pulley equals $(M + 4m)g$	F	5. Because the system is accelerated, the tension in S is smaller than $(M + 4m)g$

4.2.1.2.C. The Atwood Machine in this figure has two masses, A and B, which equal 0.45M and 0.55M respectively. The pulley is hanging from the ceiling by support S, and both the friction and the mass of the pulley are small enough to be neglected.



Test Statement	T/F	Because
1. If A and B are held at rest and then let free, B having a greater mass will get a larger acceleration than A.	F	1. A and B are connected by a string and always have the same magnitude of acceleration
2. If A and B are held at rest and then let free, then after some time they will move with constant velocity	F	2. A and B will move with constant acceleration.
3. When A and B are moving freely, then the tension in the support S is smaller than *Mg*.	T	3. When the system is moving freely, The heavier mass is accelerated downwards, and the tension in S is smaller than *Mg*
4. If the mass of A were to be decreased with *0.05M*, then the acceleration of A and B would be larger than in the original situation	T	4. The difference in mass increases and the total mass decreases, so the acceleration increases
5. If the mass of B were to be increased with *0.05M*, then the acceleration of A and B would be smaller than in the original situation	F	5. Again the difference in mass increases, and the increase in total mass is too small to change this effect, so the acceleration increases.

4.4 Appendix

"Misleading Concepts" in Mechanics.

Research has shown that the following intuitively reasonable but misleading concepts play an important role in difficulties with understanding Newtonian Mechanics[1]:

1. No forces act on a body at rest.
2. Force is needed to maintain constant velocity
3. Heavier objects fall faster

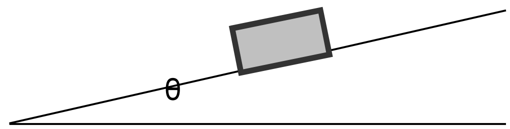
Behind these alternative concepts lies a still more fundamental lack of insight: Many students do not see the concept of force as an interaction between two bodies or two systems. Instead a force is seen as a property of the body - the impetus idea. The source of this misconception is the lack of a stringent definition of force in many schoolbooks. Without a clear force concept (*Newton's 3rd law*!) students cannot understand the fundamental laws of Mechanics.

1. Without being able to use the concept of force, students do not distinguish between the situations "no forces acting on the body" and "no net force acting on the body".
2. Alternative concept 2 is based on everyday experiences: if you do not keep pedalling your bike, it slows down. This type of experience combined with lack of understanding of friction forces leads to the conclusion "In order to keep up speed one needs a force", which, incorrectly, is converted to "Force produces constant velocity".
3. Again, observations from everyday life combined with lack of understanding of air resistance lead to this alternative concept.

The skill of analyzing the combination of forces acting on a body is dependent on understanding the concept of force. Without such an analysis it is impossible to get a picture of the dynamics of the situation.

[1] Planinic, M., Boone, W.J., Krsnik, R., & Beilfuss, M.L. (2006). Exploring alternative conceptions from Newtonian dynamics and simple DC circuits: Links between intem difficulty and item confidence. *Journal of Research in Science Teaching 43(2) pp. 150 – 171.*

4.4. A. A wooden block is lying at rest on an inclined plane with inclination θ.



Test Statement	T/F	Because
1. There are no forces acting on the block.	F	1. Gravity, friction and a normal force act on the block.
2. There is a force by the block on the plane, parallel to the plane and directed downwards	T	2. Friction acts upwards on the block, and there is a corresponding force downwards on the plane.
3. If the block is moving with constant velocity upwards along the plane, there is no net force acting on it.	T	3. When a mass is moving with constant velocity, there is no net force acting on it.
4. If there were no friction between plane and block, and you had two blocks of mass *m* and *5 m* sliding down the plane, then the block with mass *5 m* would get a greater velocity.	F	4. The two blocks would get the same acceleration.

4.4 B. On a horizontal air track gliders move without friction

Test Statement	T/F	Because
1. Two gliders, *A* and *B*, are at rest on the air track, touching each other. There is vertical force by the rail of the track on each of the gliders	T	1. This force is equal in magnitude and opposite in direction to gravity.
2. A glider *A* with mass *m* is moving with constant speed along the track and collides with a second glider *B* with mass *10 m*. During the collision, the magnitude of the force of *A* on *B* is smaller than the magnitude of the force of *B* on *A*.	F	2. Both forces have equal magnitude and opposite directions
3. One of the gliders is set moving along the track. It will keep moving with constant velocity till it reaches the end of the track.	T	3. With no horizontal force acting on the glider, it will move with constant velocity.
4. If the two gliders were to move each on its own air track and reach the end of the track at the same moment, falling on the floor, then the glider with mass *10 m* would reach the floor first.	F	4. Both gliders would reach the floor at the same time.

4.4. C. You are driving your fast sports car along a road.

Test Statement		Because
1. You park your car on a steep hill, drawing the handbrake to keep it in place. Once the car is parked, there are no forces acting on it	F	1. Gravity, the normal force of the road on the car, and friction on the tyres act on the car.
2. You are driving along the highway, approaching a bridge crossing over it. You hit on a slippery spot and crash into a pillar supporting the bridge. The magnitude of the force of the pillar on your car equals the magnitude of the force of your car on the pillar.	T	2. The two forces are of equal magnitude and opposite direction.
3. You are driving with constant velocity up a steep hill. The force of friction from the road on the tires equals the component of gravity acting along the road	T	3. Your velocity is constant, so there is no net force along the road.
4. At the moment your car hits the pillar of the bridge, two blocks of concrete come loose from the edge of the bridge, a big one and a much smaller one. They crash down on the front of your car, the big block hitting it first.	F	4. The blocks fall with the same acceleration and hit the car at the same moment.

4.4. D. An ice-hockey puck is lying at rest on a horizontal, frictionless ice surface (and we neglect air resistance).

Test Statement	T/F	Because
1.There are no forces acting on the puck.	**F**	1. Gravity and the normal force from the ice are acting on the puck.
2. You hit the puck with your stick. The magnitude of the force of the stick on the puck is greater than the magnitude of the force of the puck on the stick.	**F**	2. The two forces are of equal magnitude and opposite direction
3. After you have hit the puck, it moves across the ice surface with constant velocity.	**T**	3. There are no horizontal forces acting on the puck, so it keeps moving with constant velocity.
4. Standing on the ice, you hold in your right hand an ice hockey puck an in your left hand a packet of 10 pucks. You stretch up your hands as far as you can reach and open	**T**	4. The single puck and the packet of 10 fall with the same acceleration and reach the ice at the same time.

them at the same moment. The single puck and the packet of pucks hit the ice at the same moment		

4.4. E. A block of wood is lying at rest on a horizontal table.

Test Statement	**T/F**	**Because**
1. There are no forces acting on the block.	**F**	1. Gravity and the normal force of the table act on the block.
2. There is a downward force of the block on the table.	**T**	2. There is a normal force by the table on the block, and an opposite force of the block on the table.
3. If you connect a string to the block and pull at the string with a constant, horizontal force, the block will move with constant velocity over the table.	**F**	3. There is a net horizontal force, so the block will accelerate.
4. A wooden block and a metal block are moving across a horizontal, frictionless table. The reach the edge of the table at the same time and fall down, hitting the floor at the same moment.	**T**	4. Both blocks fall with the same acceleration and hit the floor at the same moment.

DiaMech Chapter 5.

Conservation of Mechanical Energy

Dr. Monica Ferguson-Hessler TUE (retired)

Introduction

This is Chapter 5 of DiaMech, a test of Knowledge and Skills in Elementary Mechanics. Please read the introductory Chapter 1 before using this chapter.

Knowledge and Skills tested

Knowledge	Skills
-- Concepts of Mechanical energy: Work, Kinetic energy, Potential energy. - Types of mechanical interaction between a system of bodies and its surroundings. - Concept of Work: - Concept of Conservative and Non-conservative Force - The Law of Conservation of Mechanical Energy - Conservation of Mechanical energy in the presence of Non-conservative forces. - Concept of static friction: bodies of a system that touch and move as one. - Concept of dynamic friction: bodies of a system sliding along each other.	-- Choosing a system of bodies relevant for the application of the Law of Conservation of Mechanical energy in a given situation - Identifying forces acting on a chosen mechanical system. - Distinguishing between forces performing positive, zero, and negative work on a system. - Calculating the work performed by each of the forces acting on a system. - Relating the concept of Potential Energy to the concept of Work. - Relating work performed on a system to the kinetic energy of that system. - Distinguishing between conservative and non-conservative forces. - Applying the Law of Conservation of Mechanical Energy for a system under the influence of both conservative and non-conservative forces. - Analyzing the effect of friction on the motion of the system

Content

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5.1. Concepts tested

In all physical systems, energy is conserved, if all forms of energy are included. **Mechanical energy** is conserved as long as energy is not transformed from or into non-mechanical forms (such as heat, radiation, electric or magnetic fields, chemical energy).

In applying the **law of conservation of Mechanical Energy**, we need to define:

1. **The system to which the law is applied.** This will consist of one or more bodies; these can experience mutual interactions (no energy) and external forces.
2. **The types of mechanical interaction between the system and the surroundings.** These forces can modify the Mechanical Energy of the system; by delivering energy (doing **positive work)** or removing energy **(**doing **negative work).**
3. **The Mechanical energy of the system,** which has two components:

   Internal Potential energy, being energy stored inside the system by its own components; the **Kinetic energy,** caused by the movement of its bodies, is part of this internal potential energy.

   External Potential energy, being energy stored by the position of the system in the gravity field and other fields of force.

5.1.1. Work

When a body moves along the x-axis from A to B, and a variable force $\vec{\boldsymbol{F}}(x)$ is acting on it in the *x*-direction, the work performed by $\vec{\boldsymbol{F}}(x)$ on the body is $W_{A\to B} = \int_{x_A}^{x_B} \boldsymbol{F}_x(x)\ dx.$. $W_{A\to B}$ is a scalar, and can be positive, negative or zero. From the properties of the scalar product, it follows that $W_{A\to B} = -W_{B\to A}$.

In a more general case, the body moves along any path in a plane, and the force $\vec{F}$ can have any direction in the plane. The work performed by $\vec{F}$ is then found by integrating the scalar product of the vectors $\vec{F}$ and the elements of motion $d\vec{s}$

$$W_{A\to B} = \int_A^B dW = \int_A^B \vec{F}\,(x,y)\cdot d\vec{s} = \int_{x_A}^{x_B} F_x(x,y)\ dx + \int_{y_A}^{y_B} F_y(x,y)\ dy$$

For motion in three dimensions the same formula is valid, using the 3-dimensiional scalar product of $\vec{F}$and $d\vec{s}$..

When external forces perform work on a system, that energy can be stored by the system in two forms of Potential energy: Internal Potential Energy, and External Potential Energy, or it can be transformed into some non-mechanical form of energy such as heat.

5.1.2. Test Units

For the use of vectors, see Chapter 2.5

5.1.2. A

We often use the scalar vector product to calculate the work W that is performed by a constant force **F** when acting along a curved path from point 1 to point 2: $W_{1\to2} = \int_1^2 \mathbf{F}\cdot\mathbf{ds}$. The meaning of this formula can be explained as follows:

Test Statement	True/False	Because
1. W equals the size of the force, $\lvert\mathbf{F}\rvert$, times the linear distance from 1 to 2	**F**	1. The integral is along the curve, not the shortest distance
2. W equals the sum of a large number of small contributions, $dW = \lvert\mathbf{F}\rvert \cdot ds_F$, where ds_F the component is of ***ds*** along **F**	**T**	2. Yes. $dW = \lvert\mathbf{F}\rvert\, ds\, cos\phi = \lvert\mathbf{F}\rvert\, ds_F$

5.1.2. B

We often use the scalar vector product to calculate the work W that is performed by a constant force ***F*** when acting along a curved path from point 1 to point 2: $W_{1\to2} = \int_1^2 \boldsymbol{F}\cdot\boldsymbol{ds}$. The meaning of this formula can be explained as follows:

Test Statement	True/False	Because
1. W equals the size of the force, $\lvert\mathbf{F}\rvert$, times the total length of the path	F	1. No. The scalar product of ***F*** and **ds** contains the angle between these vectors, and this angle is not constant along the path
2. W equals the sum of a large number of small contributions, $dW = F_s \cdot ds$, where F_s the component of **F** is along the path at that point	T	2. Yes. $dW = F_s \cdot ds$

5.1.2. C

We often use the scalar vector product to calculate the work W that is performed by a constant force ***F*** when acting along a curved path from point 1 to point 2: $W_{1\to2} = \int_1^2 \boldsymbol{F} \cdot \boldsymbol{ds}$. The meaning of this formula can be explained as follows

Test Statement	True/False	Because
1. W equals the area between the curve and the x axis	F	1. The variable integrated is **s**, not x
2. W equals the sum of a large number of small contributions $dW = \lvert\boldsymbol{F}\rvert \cdot ds$, where *ds* the length is of a small step along the curve	F	2. No. The scalar product of ***F*** and **ds** contains the cosine of the angle between these vectors

5.1.2. D

We often use the scalar vector product to calculate the work W that is performed by a constant force ***F*** when acting along a curved path from point 1 to point 2: $W_{1\to2} = \int_1^2 \mathbf{F} \cdot \mathbf{ds}$. The meaning of this formula can be explained as follows:

Test Statement	True/False	Because
1. W equals the size of the force, $\lvert\mathbf{F}\rvert$, times the total distance traveled in a direction parallel to ***F***	TY	1. because both the size of ***F*** and its direction are constant
2. W equals the area between the line of the vector ***F*** and the curve from 1 to 2	F	2. Such an area is not defined

5.1.2. E

We often use the scalar vector product to calculate the work W that is performed by a constant force ***F*** when acting along a curved path from point 1 to point 2:

$W_{1\to 2} = \int_1^2 \boldsymbol{F} \cdot \boldsymbol{ds}$. The meaning of this formula can be explained as follows:

Test Statement	True/False	Because
1. W equals the scalar product of the vector ***F*** with the total displacement from point 1 to point 2	F	1. W also depends on the angle between ***F*** and each element of the path
2. W equals the size of the force, $\lvert\mathbf{F}\rvert$, times the sum of a large number of contributions of the projection of each step **ds** along the curve on the line of action of ***F***	T	2. Indeed.
3. W equals the total of the distance moved times the component of the force $\lvert\mathbf{F}\rvert$,along the line of motion	T	3. Indeed.

5.1.3. Internal Potential Energy and Kinetic energy

A mechanical system can store energy internally. One form is **Elastic Energy,** for instance by reversible deformation of its own components like a spring. Many springs are linear; for instance the force spiral springs deliver is often proportional to their change in length, as expressed by the relation Force = *k x* , where *x* is the change in length and ***k.* is called the spring constant.** The work needed to pull or push the spring from rest by that distance *x* is

$$W_{A\to B} = \int_A^B dW = \int_{x_A}^{x_B} F_x(x)\ dx. = \int_{x_A}^{x_B} kx.dx = ½kx^2$$

This potential energy is part of the total energy of the system, of which the spring is a part.

A system can also store energy internally in **non-mechanical forms**.

When forces perform work on a system, one consequence can be changes in the state of motion of the bodies of the system. The concept of **Kinetic Energy *K* of the system** is defined by considering each piece of matter within the system, with mass m_i and velocity v_i , and taking the sum of $\boldsymbol{K_i} = ½\boldsymbol{m_i v_i^2}$**,** for all the pieces. The Kinetic energy of the bodies in a system is part of the Internal Potential energy of the system.

5.1.4. External Potential Energy

In many situations, the force field acting on a body is such that the work performed by the force $\vec{F}$ while it is moving from A to B is **independent of the path followed**. Gravity is an example. So if the body moves from A to B and back along any path, then the total work performed is zero. Such a force is a **conservative force.** Other types of force are **non-conservative** or **dissipative** forces, where the work performed does depend on the specific path and direction followed.

For **conservative forces only**, it is possible to introduce a function, the **potential energy** function, $U(\vec{r})$, which describes the energy state of a body that is subject to the force field $\vec{F}$, e.g. a gravitational force, then the work performed by $\vec{F}_{ext}$ in moving the body from 1 to 2 is $W_{1\to 2}$, and we define the potential energy function $U(\vec{r})$ by

$W_{1\to 2} = \int_1 \int_1^2 \vec{F} \cdot d\vec{s} = U(\vec{r}_2) - U(\vec{r}_1)$. Note to editors: remove duplicate integral.

The coordinate or level where U = 0 is chosen for the relevant system, and so the potential energy for each point in the system is defined.

5.**2. Conservation of Mechanical Energy**

In real mechanical systems there are always non-conservative forces, and Mechanical Energy is not conserved. In many real situations, however, non-conservative processes play only a minor role and can in first approximation be neglected. Those simplified representations do obey the law of Conservation of Mechanical Energy. They are very useful for understanding and solving a wide variety of problems in Mechanics.

5.2.1. Situations with conservative forces only

As is the case in the application of other conservation laws, the first step in applying the Law of Conservation of Mechanical Energy is the **definition of the system that is going to be used.**

The total energy of a system consists of its kinetic energy *K,* its external potential energy U_e and its internal potential energy U_i. When only conservative forces are acting on and in the system, the law of Conservation of Mechanical Energy can he be written as

$$E_{tot} = K_1 + U_{g1} + U_{i1} = K_2 + U_{g2} + U_{i2}$$

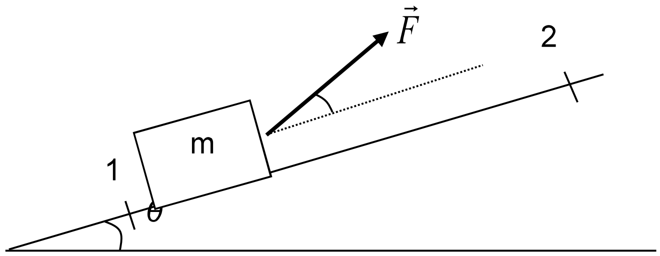
which states that

1. The total energy of the system consists of its kinetic energy *K,* its external potential energy U_e and its internal potential energy U_i .
2. The total energy of the system in state 1 equals the total energy in state 2. The mechanical energy is conserved.

5.2.2 Test Units

5.2.2. A

A wooden block of mass *m* is being pulled up a frictionless sloping plane over a distance *d*, from 1 to 2, by a force $\vec{F}$. The inclination of the plane is *θ* and $\vec{F}$ makes and angle α with the plane.

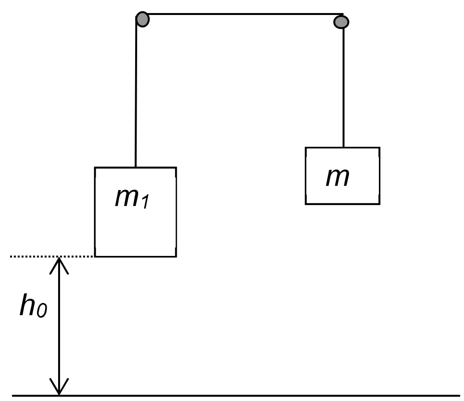


Test Statement	T/F	Because
1. If *m* is pulled up the plane from 1 to 2 by $\vec{F}$, it gains gravitational potential energy.	**T**	1. Work is done against gravity, and the potential energy increases.
2. According to the definition, the potential energy of *m* at 1 is zero.	**F**	2. The zero level of the potential energy has not been is defined .
3. If *m* is pulled up from 1 to 2 by $\vec{F}$ and then let free to slide down to 1, the total change of potential energy is zero.	**T**	3. The potential energy gained on the way up is lost when *m* is sliding down to the starting point.
4. If *F* pulls *m* up from position 1 to 2, then the gain in potential energy is greater than *mgd sin θ*	**F**	4. The potential energy depends only on the vertical displacement

5.2.2 B.

Two masses, m_1 and m_2 (which is smaller) are connected by a string, running over two frictionless pegs (see figure); m_1 is supported at a height h_0 above the floor. We consider a system consisting of the two masses. Gravitational potential energy is counted as zero at the level of the floor.

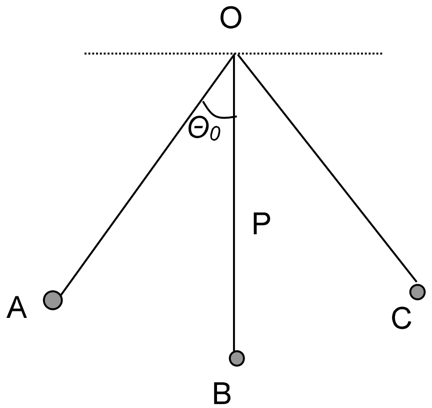
Mass m_1 is let free.



Test Statement	T/F	Because
1. When m_1 hits the floor, all potential energy of the system has been converted to kinetic energy.	**F**	1. m_2 has gained potential energy while m_1 is moving down.
2. While m_1 is moving, the kinetic energy of m_1 is larger than the kinetic energy of m_2.	**T**	2. The speeds are the same and the mass of m_1 is larger.
3. m_1 will hit the floor with a velocity $v_1 = \sqrt{2gh_0}$	**F**	3. The loss of potential energy is converted into kinetic energy of **both** masses.
4. If m_2 is given an initial speed v_0 downwards at the moment the masses are let free, then m1 will hit the floor with a smaller speed than when m_1 is just let free to move.	**F**	4. The system will start with a larger total energy and therefore have a larger kinetic energy at the moment m_1 hits the floor.

5.2.2..C

A pendulum of length ℓ is hung from point O. The mass of the string is small and is neglected. The mass is pulled out to point A, where the pendulum makes an angle θ_0 with the vertical, held at rest, and then let free to swing in a vertical plane.

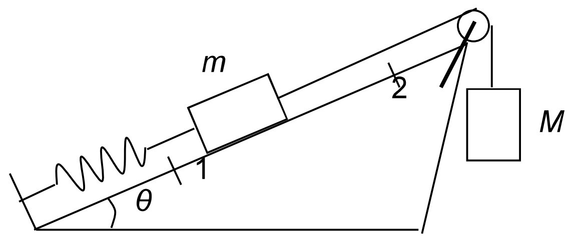


The zero level for gravitational potential energy is a plane through O.

Test Statement	T/F	Because
1. The total energy of the mass is negative through the whole motion	**T**	1. At A the only form of energy is negative potential energy.
2. When the mass passes B, it has kinetic energy but no potential energy	**F**	2. At B the mass has a negative potential energy
3. The mass will stop at a point C, which is at same level as A	**T**	3. At this level all its kinetic energy has been converted to potential energy.
4. If you want the mass to reach a level higher than C, then you can give it an initial velocity swinging downwards at A	**T**	4. The total energy is now greater than in the first case.

5.2.2 D

A mass *m* can move across a frictionless plane with inclination *θ*. The lower end of *m* is attached to a fixed spring, and from its upper end a string runs over a weightless and frictionless pulley to a second mass, *M* (see figure). The spring constant is *k*. Two points, 1 and 2 are marked on the plane, the distance between them is *d*. When *m* is at point 1, the spring is exerting no force on *m*.



Test Statement	T/F	Because
1. If *m* moves from 1 to 2, then *M* experiences a loss of potential energy of *Mgd*.	T	1. *M* moves down the same distance as *m* moves along the plane.
2. If *m* moves from 1 to 2. then *m* experiences a gain of gravitational potential energy of *mgd*	F	2. Constant acceleration, slowing down
3. If *m* moves from 1 to 2. then the potential energy of the spring increases by ½ kd^2	T	3. The spring is stretched from zero to *d*; its potential energy increases by ½kd^2
4. If *m* is forced to move a short distance down the plane from point 1, then the change in potential energy of the spring would be negative.	F	4. The spring would increase its potential energy when being compressed.

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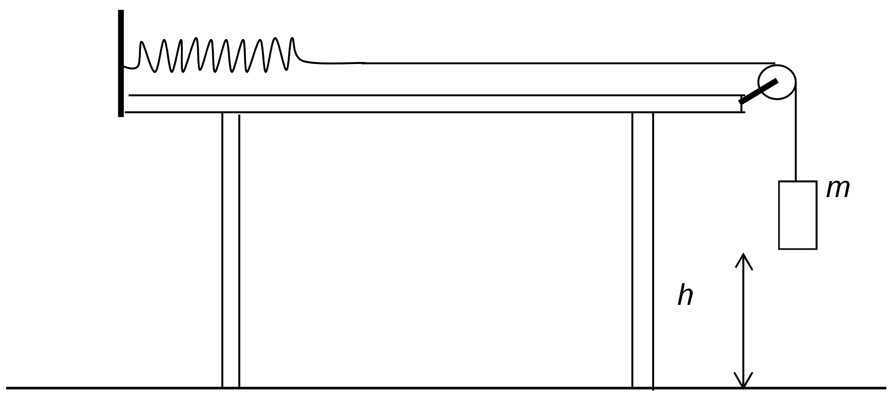
5.2.2. E.

A horizontal string, running over a pulley, connects the end of a spring to a block of mass, *m*, which is hanging on a vertical chapter of the string.. The mass of the pulley is small and is ignored. The gravitational potential energy *U = 0* at the level of the floor.

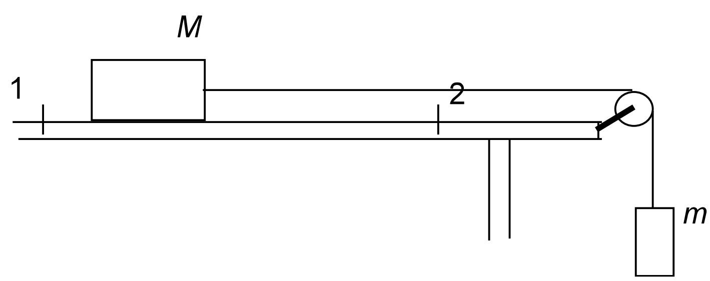
Mass *m* is held at a height *h* above the floor and then let free to move downwards. Just before *m* reaches the floor its velocity is zero.



Test Statement	T/F	Because
1. Having lost an amount of gravitational potential energy *mgh* the mass comes to rest just above the floor	F	1. mg*h* has been converted to potential energy of the spring, and *m* moves upwards again.
2. The spring constant is $\frac{2mg}{h}$	T	2. When *m* is just above the floor, the potential energy of the spring equals $\frac{1}{2} kh^2 = mgh$.
3. While *m* moves down a small distance Δx the kinetic energy of *m* increases by $\Delta K = mg\,\Delta x$	F	3. Some of the gravitational potential energy goes into elastic potential energy of the spring.
4. While m moves down its kinetic energy first increases and then decreases.	T	4. From height h to ½ h K increases, and from height ½ h to zero K decreases again.

5.2.2 F.

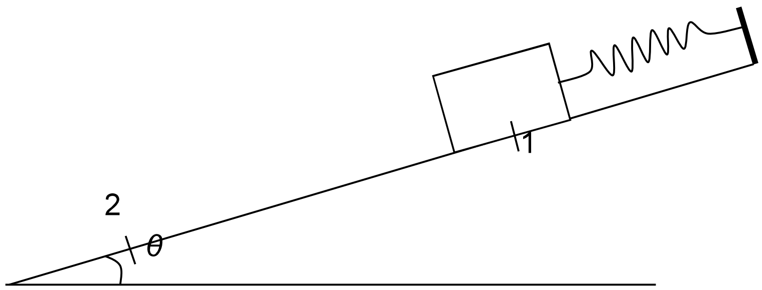
A wooden block of mass *M* is moving across a frictionless horizontal table over a distance *d*. from 1 to 2. A string, running over a pulley, connects the block to another mass, *m*, which is hanging on a vertical chapter of the string. The mass of the pulley and string is ignored. We consider a system consisting of the two masses connected by the string..



Test Statement	**T/F**	**Because**
1. When *M* moves from 1 to 2 the system experiences a loss of potential energy.	T	1. The system gains an equal amount of kinetic energy, energy is conserved.
2. When *m* hits the floor, all the kinetic energy will be lost.	F	2. The mass *m* stops, but the mass M keeps its kinetic energy and slides on at constant speed.
3. The magnitude of the loss of potential energy as *M* moves from 1 to 2 is *(M + m) gd.*	F	3. *M* does not contribute to the loss of potential energy.
4. If there were sme friction between M and the table the loss of gravitational potential energy would be smaller.	F	4. Friction does not influence gravitational potential energy.

5.2.2. G

A wooden block of mass *m* is held in rest in position 1 on a frictionless inclined plane with inclination *θ.* The topside of the block is connected to a fixed spring with spring constant *k* (see figure). When the block is at 1, the force of the spring on the block is zero. The block is let free and moves down the inclined plane and passes position 2 after having moved through a distance *d*.



Test Statement	T/F	Because
1. While moving from position 1 to position 2, *m* experiences a loss of gravitational potential energy which is wholly compensated by a gain of kinetic energy.	T	1. There is no energy loss, energy is conserved.
2. The block comes to rest and stays at position 2 if $d = \dfrac{2mg\sin\theta}{k}$	F	2. The block comes to a stop in 2, but is not in equilibrium. It will move up again and oscillate between positions 1 and 2.
3. When *m* passes position 2, the total energy of the system of spring and mass is smaller than it was at position 1	F	3. There is no energy loss.

5.3. Situations with both conservative and non-conservative forces

Let us mention a few common forms of non-conservative forces in mechanical systems. External force can be **non-conservative** or **dissipative** forces, where the work performed does depend on the specific path followed. Mechanical energy can be lost by irreversible changes within the system like friction or collisions. An external force is a force between the surroundings and the system.

5.3.1. Friction

Friction is a force that **counteracts relative motion between two objects in contact**, e.g. an object resting on or sliding over any surface. Friction occurs in all mechanical systems wherever bodies of the system are in contact with each other or with the surroundings. Friction is very useful: without friction with the road, you cannot walk up a hill!

We need to distinguish two forms of friction: *when the bodies have no relative motion*, which we call **static friction**, *and when they have such motion*, which we call **dynamic friction.**

Static Friction.

The bodies that are in contact remain at fixed positions relative to each other (i.e. no sliding). The points of contact transmit both normal and tangential forces. When they move together, the work done by the first surface on the second and by the second on the first add to zero (same displacement, equal and opposite forces). With **Static Friction the system of forces remains conservative, and Mechanical Energy is conserved**. . .

If two surfaces touch with a normal force N, and F_s is the greatest tangential force that can be transmitted without causing slipping, then $\mu_s = F_s/N$ is called the **coefficient of static friction** between the surfaces. This can vary greatly depending on the texture of the surfaces, the pressure applied, and the presence of lubricants.

Dynamic Friction

When two objects touch and the tangential force becomes too great, they will start to slip at the point of contact. If the normal force is N and the tangential force is F_d, we can similarly define $\mu_d = F_d/N$, the **coefficient of dynamic friction.** This is only a constant by approximation. Both coefficients of Friction, μ_s and μ_d depend on the form and the materials of the surfaces; and can depend on N, and are very dependent on lubricants. In most cases, $\mu_d < \mu_s$.

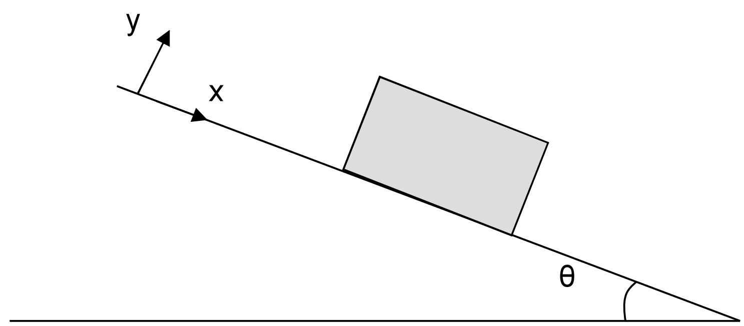
In dynamic friction the forces on the two surfaces are still equal and opposite, but the displacements differ, and the work delivered to the two surfaces at the point of contact are no longer equal and opposite. The two sliding objects deliver a net flow of energy to the point of contact. Mechanical energy is lost (transformed to heat).

5.3.1. Dynamic Frictional forces and Newton’s 2nd law

Friction and Newton’s 1st and 2nd laws .The direction and magnitude of frictional forces

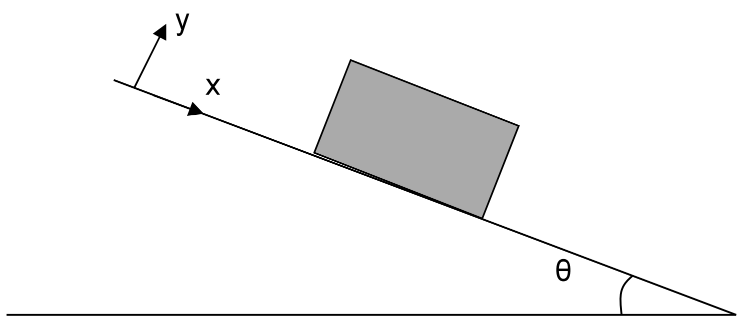
Stress that these frictional forces are all dynamic (sliding)

5.3.1.A. A polished wooden block is resting on an inclined plane also made from polished wood. The coefficient of static friction between the block and the plane is *0.4* and *θ* is small enough to let the block remain at rest.



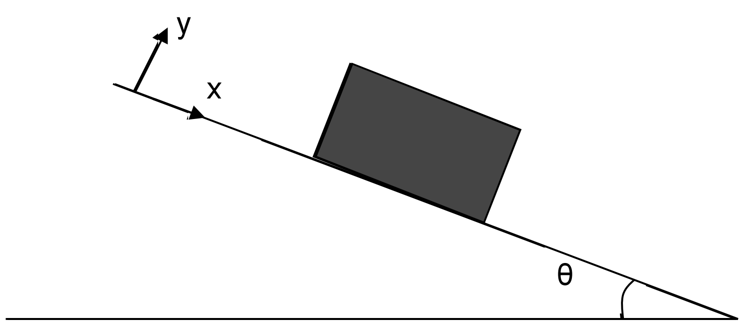
Test Statement	T/Y	Because
1. There is a frictional force acting on the block in the negative x-direction	T	1. The block will tend to slide downwards, so the friction force is directed upwards.
2. If *θ* is increased, the block will start moving with constant velocity down the slope.	F	2. Once the block starts moving in will move with increasing velocity.
3. If *θ* is increased, it is possible to find a value of *θ* for which the block once set moving, will continue moving with a constant velocity down the slope.	T	3. If the dynamic friction just compensates the component of gravity in the x-direction
4. The value of *θ* that makes the block start sliding is greater than the value that makes the block move with a constant velocity	T	4. The value of the coefficient of dynamic friction is smaller than that of static friction.

5.3.1.B. A polished wooden block is resting on an inclined plane also made from polished wood. The coefficient of static friction between the block and the plane is *0.4* and θ is small enough to let the block remain at rest.



Test Statement	T/F	Because
1. As long as the block remains at rest, there are only two forces acting on it, gravity and the normal force from the plane.	F	1. There is also a friction force needed to keep the block at rest.
2. There is a friction force acting on the plane in the negative x-direction	F	2. The block will tend to slide downwards, so the friction force of the block on the plane is directed downwards.
3. By finding the angle θ that makes the block move down the slope with a constant velocity it is possible to measure the coefficient of dynamic friction.	T	3. If the dynamic friction just compensates the component of gravity in the x-direction then the acceleration will be zero.
4. The value of θ that makes the block start sliding is greater than the value that makes the block move with a constant velocity	T	4. The value of the coefficient of dynamic friction is smaller than that of static friction.

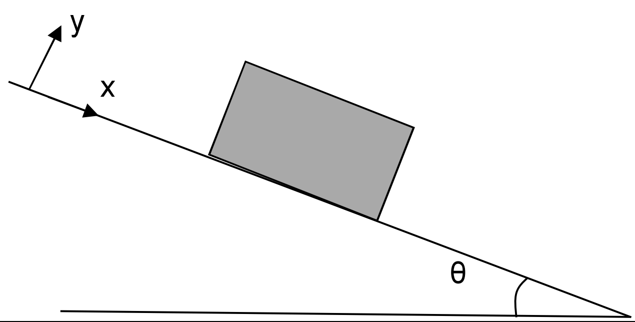
5.3.1.C. A polished wooden block with mass *M* is resting on an inclined plane also made from polished wood. The coefficient of static friction between the block and the plane is *0.4* and θ is small enough to let the block remain at rest.



Test Statement	T/F	Because
1. If the mass *M* were to be increased and all other properties of the system to remain the same, then for a certain value of *M* the block would start sliding down the plane.	F	1. If all other properties remain the same, the block would remain at rest, independently of *M*.
2. If θ is increased, the force of friction on the block will increase in magnitude	T	2. This is correct within certain limits. The frictional force will increase to the maximum of static friction
3. By finding the angle θ that just makes the block start moving it is possible to measure the coefficient of static friction	T	3. The static friction has then reached its maximum value and equals the component of gravity along the plane.
4. If θ is large enough to let the block move down the slope, then the friction on the block keeps increasing with θ.	F	4. The dynamic friction decreases as θ increases because the normal force decreases..

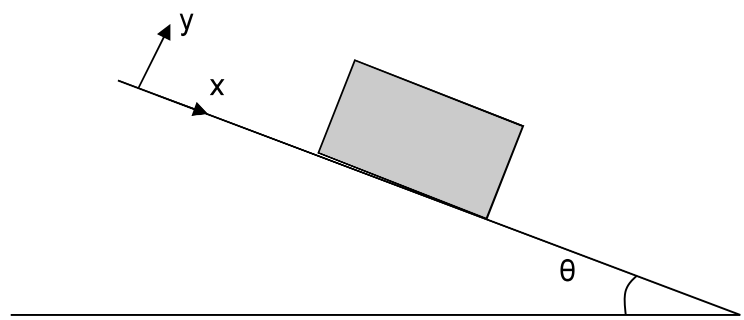
5.3.1.D. A polished wooden block with mass *M* is resting on an inclined plane also made from polished wood. The coefficient of static friction between the block and the plane is *0.4* and θ is small enough to let the block remain at rest.

Because



Test Statement	T/F	Because
1. If the block is turned around and placed on the plane on its smallest surface, then it might start sliding down (assuming that it does not tip over)	F	1. If the block is at rest on one surface, it is also at rest on other surfaces
2. If the block (in its original position) is set moving up the plane, it will slow down, come to rest, and then return to its starting point.	F	2. It will come to rest at a higher position.
3. If you measure the angle θ at which the block starts to slide, then the result is independent of which surface the block is resting on.	T	3. The friction is independent of the size of the surface the block is resting in.
4. The force of friction between the plane and the block is independent of the property of the surface of the plane as long as the surface of the block remains the same	F	4. The force of friction unique for each pair of surfaces being in contact.

5.3.1.E. A polished wooden block with mass *M* is resting on an inclined plane also made from polished wood. The coefficient of static friction between the block and the plane is *0.4* and θ is small enough to let the block remain at rest.



Test Statement	T/F	Because
1. If the block is turned around and placed on its smallest surface, then the frictional force on the block will become smaller.	F	1. The force of friction remains the same.
2. If the block is set moving down the slope, then it is possible that it comes to rest at a lower position on the plane.	T	2. If θ is small the dynamic friction being larger than the component of gravity along the slope, will slow down and stop the block.
3. If the block is set moving up the slope, then there is a force of friction on the block in the x-direction	T	3. The friction opposes the motion upwards on the plane.
4. The force of friction on the block when it is moving down the slope equals the maximum value that the friction from the slope on the block can reach when the block is at rest.	F	4. The coefficient of dynamic friction is smaller than the coefficient of static friction, which determines the maximum of the static force of friction.

5.3.2. Collisions

When bodies in a system collide, they exert forces on each other for a short moment. For some materials like billiard balls, we can often use the Law of Conservation of Mechanical Energy as a good approximation, as very little mechanical energy is lost (i.e. turned into heat). The same will not apply to tennis balls, and certainly not to balls of dough or snowballs.

5.3.3. The extended form of the Law

When non-conservative forces act on or in a system, we must replace the Law of Conservation of Mechanical energy to include the work performed by these forces. The extended form of this law then becomes:

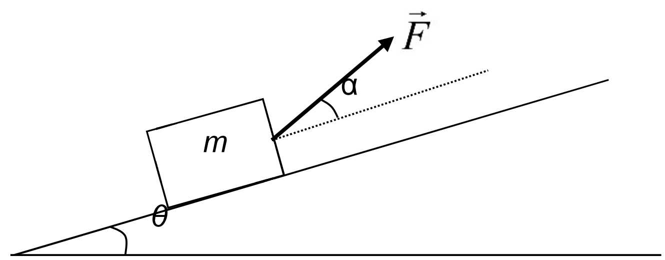
$$K_1 + U_{e1} + U_{i1} + \sum_i W_{i\ 1\to 2} = K_2 + U_{e2} + U_{i2}$$

which states that the total energy of the system in state 1, plus the (negative) work performed by the non-conservative forces during the transition to state 2, equals the total energy in state 2. **The network by friction on the system of two surfaces together is always negative!** Energy lost by friction reappears as heat.

5.3.4. Test Units

5.3.4. A.

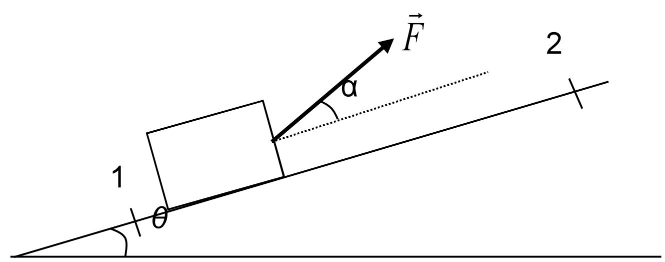
A wooden block of mass *m* is being pulled up a sloping plane for a distance *d* by a force $\vec{F}$. The inclination of the plane is θ and $\vec{F}$ makes and angle α with the plane. The coefficient of dynamic friction between the block and the plane is μ_d.



Test Statement	T/F	Because
1. The work performed by $\vec{F}$ on *m* is $F \cos \alpha \, d$.	T	1. The component of $\vec{F}$ parallel to the plane, times the distance *d*, is the work done on the block.
2. Because the block is being pulled upwards, gravity performs no work on it.	F	2. Gravity performs negative work on the block.
3. The work performed on the block by friction is $-\mu_d (mg \cos \theta - F \sin \alpha)\, d$.	T	3. The frictional force is μ_d times the normal force on the plane.
4. The work performed on the block by the normal force is $mg \cos \theta \, d \sin \theta$.	F	4. The normal force on the block performs no work.

5.3.4. B

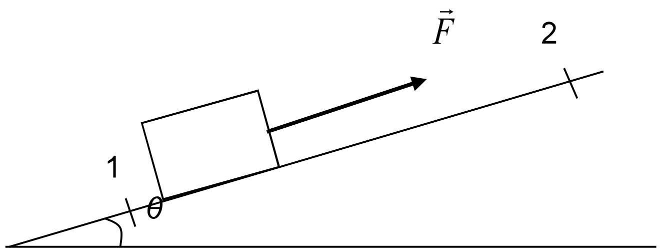
A wooden block of mass m is being pulled up a sloping plane over a distance d, from 1 to 2, by a force $\vec{F}$. The inclination of the plane is θ and $\vec{F}$ makes and angle α with the plane. The coefficient of dynamic friction between the block and the plane is μ_k. The velocities at the two points 1 and 2 are v_1 and v_2 respectively and the kinetic energies are K_1 and K_2.



Test Statement	T/F	Because
1. $\Delta K = K_2 - K_1$ may be positive, zero or negative.	T	1. Both positive and negative work is done on the block and the sum may be positive, zero or negative.
2. If $\mu_k = 0$ then ΔK is always positive.	F	2 . Gravity also performs negative work on the block
3. If the slope becomes steeper and everything else remains the same, then ΔK decreases.	T	3. More work has to be done against gravity, and ΔK decreases.
4. If K2 = 0, then K1 has been wholly converted to heat.	F	4. Part of K1 was used to compensate negative work by gravity

5.3.4. C

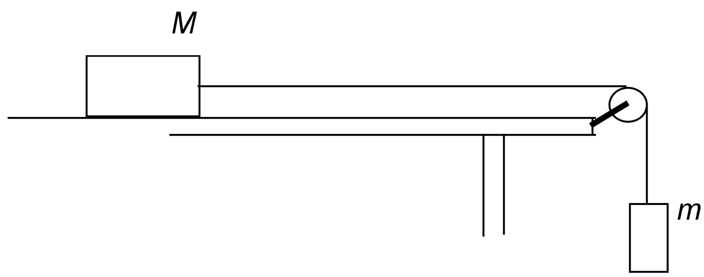
A wooden block of mass *m* is being pulled up a sloping plane over a distance *d*, passing through position 1 and position 2, by a force $\vec{F}$, acting parallel to the plane. The inclination of the plane is *θ*. The kinetic energy at 1 and 2 is K_1 and K_2 respectively. There is a coefficient of dynamic friction μ_k between the block and the plane.



Test Statement	T/F	Because
1. The gain in potential energy when *m* moves from 1 to 2 is wholly compensated by a loss of kinetic energy $K_2 - K_1$.	F	1. External work is done by $\vec{F}$ and by friction.
2. The total work on *m* by the external non-conservative forces is > 0	F	2. Friction gives a negative contribution to the work, which may make W negative.
3. If $K_1 = 0$ then the total work on *m* by the external non-conservative forces is > 0.	T	3. Gravitational potential energy increases, so positive work must be performed on *m*
4. Whether m moves from 1 to 2 or from 2 to 1, it is always possible to adjust in such a way that K1 = K2.	T	4. In both cases can be chosen to compensate the work done by gravity and friction

5.3.4. D

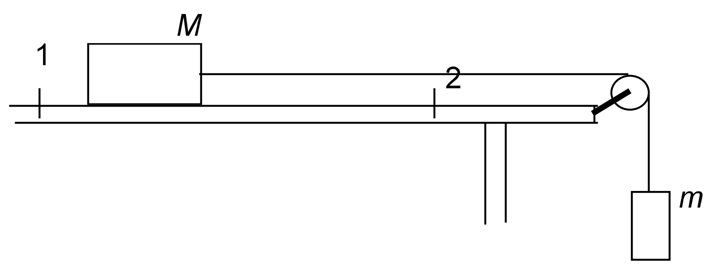
A wooden block of mass *M,* lying on a horizontal table, is attached to a string that runs over a pulley, and has another mass, *m*, hanging on its loose end. μ_d is the coefficient of dynamic friction *between* the block *M* and the surface of the table. The mass and friction of the pulley and sting can be neglected. We consider a system consisting of the two masses connected by the string. At the start *M* is held fast. When released, the block *M* moves by a distance d



Test Statement	T/F	Because
1. If the block is moving towards the pulley, then gravity performs positive work on the system.	T	1. The hanging block is moving in the direction of gravity.
2.The work done by dynamic friction $is - \mu_d$ *mg d.*	T	2. The force of dynamic friction does not depend on the speed
3. If the system is accelerated, the of the work of friction on the system is smaller than in the statement above.	F	3. Acceleration does not influence the work done by friction.
4. The tension in the string performs positive work on the system	F	4. The tension in the string is an internal force in the system.

5.3.4. E

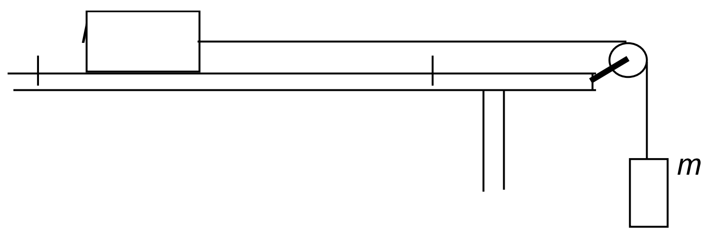
A wooden block of mass M is moving across a horizontal table over a distance d. from 1 to 2. A string, running over a pulley, connects the block to another mass, m, which is hanging on a vertical chapter of the string. There is a coefficient of dynamic friction μ_k between the block and the surface of the table. The mass of the pulley is small and is neglected. We consider a system consisting of the two masses connected by the (massless) string. At point 1 M has speed v_1 and the system has a kinetic energy K_1, at 2 the speed of M is v_2 and the kinetic energy of the system is K_2.



Test Statement	T/F	Because
1. $\Delta K = K_2 - K_1$ is always > 0.	F	1. If μ_k is large, ΔK may be negative.
2. $\Delta K = mg(1-\mu_k)d$	T	2. Gravity does work mgd and friction negative work $mg\,\mu_k\,d$.
3. If $\mu_k = 0$ and $v_1 = 0$, then $v_2^2 = \frac{2}{M} mgd$	F	3. The kinetic energy is shared by m and M
4. If M were moving from 2 to 1, then ΔK would always be < 0.	T	4. Gravity and friction would perform negative work on the system.

5.3.4. F

A wooden block of mass *M* is moving across a horizontal table over a distance *d*. from 1 to 2. A string, running over a pulley, connects the block to another mass, *m*, hanging on the end of the string. There is a coefficient of dynamic friction μ_k between the block and the surface of the table. The mass and friction of the pulley and string may be ignored. We consider a system consisting of the two masses connected by the (mass less) string..

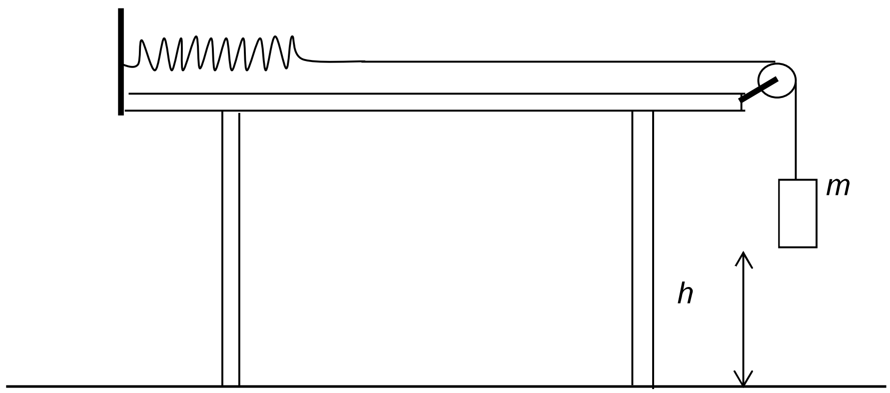


Test Statement	**T/F**	**Because**
1. When *M* moves from 1 to 2 the system experiences a loss of potential energy due to friction.	F	1. Potential energy is lost, but that is not due to Friction
2. When *M* moves from 1 to 2 the system experiences a loss of gravitational potential energy.	T	2. Mass *m* moves down and loses potential energy.
3. The magnitude of the loss of potential energy as *M* moves from 1 to 2 is $(M + m)gd$.	F	3. M does not contribute to the loss of potential energy.
4. If there were no friction the loss of gravitational potential energy from 1 to 2 would be smaller.	F	4. Friction does NOT influence gravitational potential energy.

5.3.4. G

A horizontal string, running over a pulley, connects the end of a spring to a block of mass, *m*, which is hanging on a vertical chapter of the string. The mass of the pulley is small and is ignored. The gravitational potential energy $U = 0$ at the level of the floor.

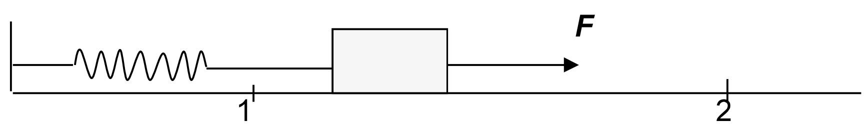
The mass *m* is held at a height *h* above the floor and then let free to move downwards. Just before *m* reaches the floor its velocity is zero.



Test Statement	T/F	Because
1. Having lost an amount of gravitational potential energy *mgh* the mass comes to rest just above the floor	F	1. mg*h* has been converted to potential energy of the spring, and *m* moves upwards again.
2. The spring constant is $\frac{2mg}{h}$	T	2. When *m* is just above the floor, the potential energy of the spring equals $\frac{1}{2} kh^2 = mgh$.
3. While *m* moves down a small distance Δx the kinetic energy of *m* increases by $\Delta K = mg\,\Delta x$	F	3. Some of the gravitational potential energy goes into elastic potential energy of the spring.
4. While m moves down its kinetic energy first increases and then decreases.	T	4. From height h to ½ h K increases, and from height ½ h to zero K decreases again.

5.3.4. H

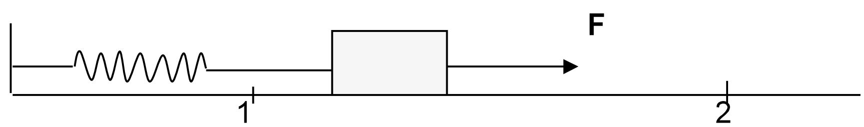
A wooden block with mass m is moving across a horizontal table from 1 to 2, over a distance d. The coefficient of dynamic friction between the table surface and the block is μ_k. One side of the block is connected to a spring with spring constant k. When the block is at 1, the force of the spring on the block is zero. A constant horizontal force $\vec{F}$ acts on the other side of the block (see figure). When the block passes 1 its speed is v_1 and its kinetic energy K_1, and when it passes 2 it has speed v_2 and kinetic energy K_2.



Test Statement	T/F	Because
1. $\Delta K = K_2 - K_1$ can be positive, zero, or negative	T	1. The sum of the work performed by the forces acting on the block may be positive, zero or negative
2. If $\mu_k = 0$, then v_2 is always larger than v_1.	F	2. v_2 may be smaller if k is relatively large.
3. If $F\ell < \frac{1}{2} k\ell^2 + \mu_k mg\, \ell$ and $v_1 = 0$, then $K_2 < 0$	F	3. K is always positive. In this situation the block never reaches point 2

5.3.4. I

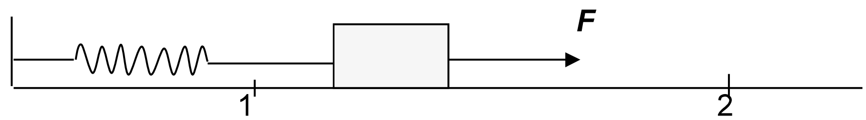
A wooden block with mass *m* is moving across a horizontal table starting from rest at position 1 moving to position 2, a distance *d*. One side of the block is connected to a spring with spring constant *k*. A constant horizontal force $\vec{F}$ acts on the other side of the block (see figure). The coefficient of dynamic friction between the table surface and the block is μ_k. When the block is at 1, the force of the spring on the block is zero. When the block is at 2, the force of the spring on the block equals. $\vec{F}$.



Test Statement	T/F	Because
1. The net work performed by $\vec{F}$ while *m* is moving from 1 to 2 is wholly converted into kinetic energy of *m*.	F	1. Part of this work is converted into other types of energy.
2. When *m* passes position 2, its kinetic energy equals $Fd - \mu_k.mg\ d$.	F	2. Part of the work performed on the block is converted to elastic potential energy of the spring.
3. The larger the coefficient of dynamic friction, the smaller the kinetic energy K_2 at the moment *m* passes through 2.	T	3. The larger the work of friction, the less energy can be converted into kinetic energy.
4. The larger the coefficient of dynamic friction, the smaller the elastic potential energy of the spring at the moment m passes 2.	F	4. The elastic potential energy of the spring only depends on the displacement d and the spring constant k.

5.3.4. J

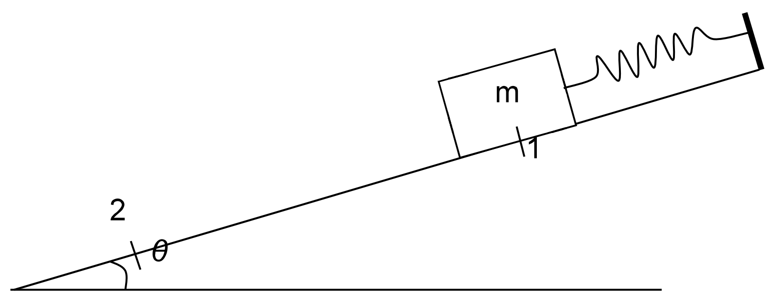
A wooden block with mass *m* is moving across a horizontal table from 1 to 2, a distance *d*. The coefficient of dynamic friction between the table surface and the block is μ_k. One side of the block is connected to a spring with spring constant *k*. When the block is at 1, the force of the spring on the block is zero. A constant horizontal force $\vec{F}$ acts on the other side of the block (see figure).



Test Statement	T/F	Because
1. While the block is moving from 1 to 2 the spring does positive work on it.	F	1. The spring does negative work on the block.
2. While the block is moving from 1 to 2 $\vec{F}$ does positive work *Fd* on the block	T	2. $\vec{F}$ is constant and along the displacement *d*.
3. If F gradually decreases to zero and the block moves from 2 to 1, then the work of the spring on the block is positive	T	3. Then the force of the spring is along the displacement.
4. If F gradually decreases to zero and the block moves from 2 to 1, then the work of friction on the block would be positive.	F	4. The work of friction is always negative.

5.3.4. K

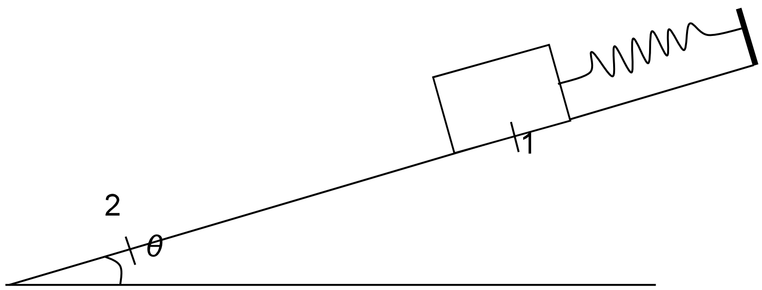
A wooden block of mass *m* is held in rest in position 1 on an inclined plane with inclination *θ.* The topside of the block is connected to a fixed spring with spring constant *k* (see figure). The coefficient of dynamic friction between the surface of the plane and the block is μ_k. When the block is at 1, the force of the spring on the block is zero. The block is let free and moves down the inclined plane and passes position 2 after having moved through a distance *d*.



Test Statement	T/F	Because
1. While moving from 1 to 2, *m* loses gravitational potential energy and gains the same amount of kinetic energy.	F	1. Energy is not conserved; due to friction, and the spring stores energy too.
2 If there is no friction between *m* and the plane, then the block comes to rest at position 2 if ***d = 2 mg sin(θ) / k***	T	2. In this case all the gravitational potential energy has been converted into elastic potential energy and the kinetic energy is zero.
3.If there is friction and m would be let free in position 2, and k is large enough to make the block move up the slope, then the total energy of the system would be increasing while m is moving from 2 to 1.	T	3. As always dynamic friction can only perform negative work on the system

5.3.4. L

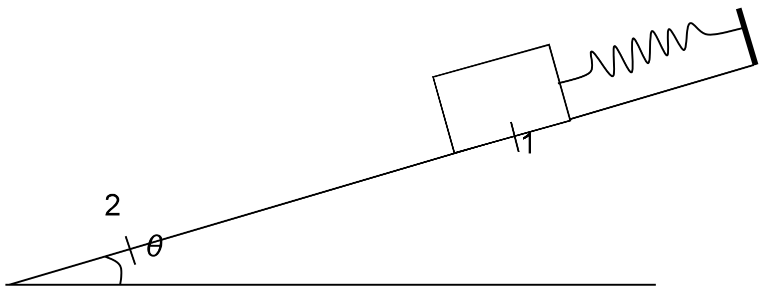
A wooden block of mass *m* is held in rest in position 1 on an inclined plane with inclination *θ*. The topside of the block is connected to a fixed spring with spring constant *k* (see figure). The coefficient of dynamic friction between the surface of the plane and the block is μ_k. When the block is at 1, the force of the spring on the block is zero. The block is let free and moves down the inclined plane and passes position 2 after having moved through a distance *d*.



Test Statement	T/F	Because
1. While moving from position 1 to position 2, *m* experiences a loss of gravitational potential energy which is wholly compensated by a gain of kinetic energy.	F	1. There are other forms of energy playing a role in the system considered.
2. If there is no friction between *m* and the plane, then the block comes to rest at position 2 if $d = \frac{2mg\,sin\theta}{k}$	T	2. In this case all the gravitational potential energy has been converted into elastic potential energy and the kinetic energy is zero.
3. When *m* passes position 2, the total energy of the system of spring and mass is smaller than it was at position 1	T	3. Negative work is performed on the system by friction.
4. If m would be let free in position 2, and k is large enough to make the block move up the slope, then the total energy of the system would be increasing while m is moving from 2 to 1.	F	4. In this case friction performs negative work on the system

5.3.4. M

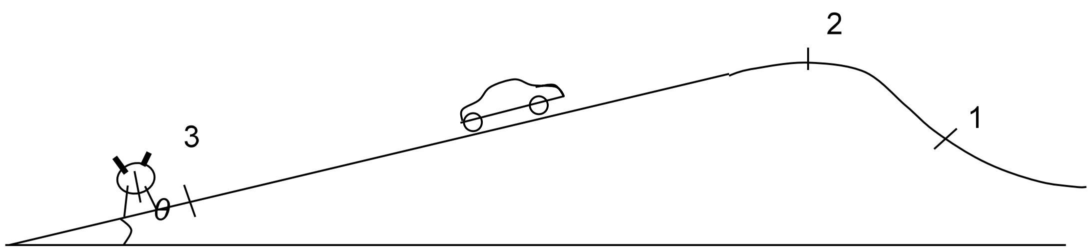
A wooden block of mass *m* is held in rest in position 1 on an inclined plane with inclination *θ*. The topside of the block is connected to a fixed spring with spring constant *k* (see figure). The coefficient of dynamic friction between the surface of the plane and the block is μ_k. When the block is at 1, the force of the spring on the block is zero. The block is let free and moves down the inclined plane and passes position 2 after having moved through a distance *d*.



Test Statement	T/F	Because
1. While moving from position 1 to position 2, *m* experiences a loss of gravitational potential energy which is wholly compensated by a gain of kinetic energy.	F	1. There are other forms of energy playing a role in the system considered.
2. If there is no friction between *m* and the plane, then the block comes to rest at position 2 if $d = \frac{2mg\, sin\theta}{k}$	T	2. In this case all the gravitational potential energy has been converted into elastic potential energy and the kinetic energy is zero.
3. When *m* passes position 2, the total energy of the system of spring and mass is smaller than it was at position 1	T	3. Negative work is performed on the system by friction.
4. If m would be let free in position 2, and k is large enough to make the block move up the slope, then the total energy of the system would be increasing while m is moving from 2 to 1.	F	4. In this case friction performs negative work on the system

5.3.4. N

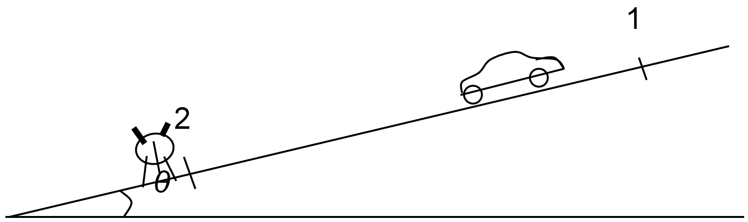
A car with mass m is driving up a hill and down a slope with inclination θ. As the driver passes point 2, he sees a herd of cows blocking the road further down at point 3 He slams on the brakes, and the car starts to skid over the wet surface of the road. The distance over the surface of the road from 2 to 3 is ℓ. The coefficient of dynamic friction between the tyres and the road surface is μ_k. The car experiences an air resistance, which is proportional to the speed.



Test Statement	**T/F**	**Because**
1. If the work of friction (air and surface) on the car compensates the kinetic energy at point 2, then the car will stop in time.	F	1. Not only kinetic energy at 2 but also loss of potential energy between 2 and 1 must be compensated.
2. The larger the speed of the car at 2, the larger the contribution of air resistance to braking.	T	2. Air resistance does more work at high speeds.
3. If the driver brakes intermittently, so that the wheels stay turning and stop slipping so much, he has a better chance of not hitting the cows.	T	3. When the tyres are not slipping, the static coefficient of friction applies, which is larger, and therefore the braking is stronger.
4. The larger the loss of potential energy between 2 and 3, the smaller the work performed by friction.	T	4. The steeper the hill, the smaller the normal force on the car and thus friction.

5.3.4. O

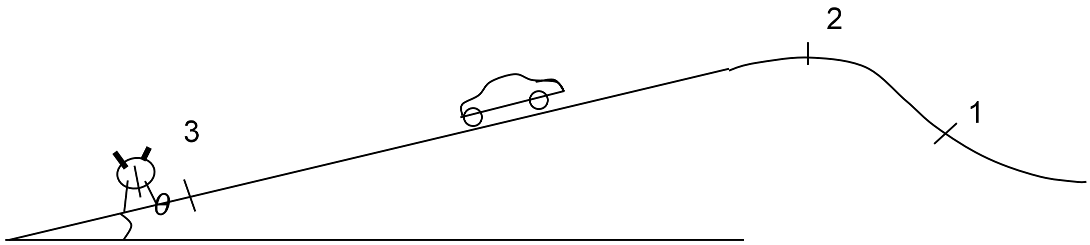
A car is driving down a slope with inclination θ. As the driver passes point 1 with a speed v_1, he sees a herd of cows blocking the road further down at point 2 at a distance ℓ. He slams on the breaks, and the car starts to skid over the wet surface of the road. The coefficient of dynamic friction between the tyres and the road surface is μ_k. The car experiences an air resistance, which is proportional to the speed. The total work of the air resistance is W_{air}. The kinetic energy of the car at 1 and 2 are K_1 and K_2 respectively



Test Statement	T/F	Because
1. To bring the car to a stop at 2 it is necessary that $\Delta K = K_2 - K_1 < 0$.	T	1. ΔK must be negative and equal $-K_1$.
2. Gravity gives a positive contribution to $\Delta K = K_2 - K_1$	T	2. Gravity acts to increase K
3. If $mg\ell \sin\theta > \mu_k\, mg \cos\theta\, l + W_{air}$ then kinetic energy of the car will be transferred to the cows	T	3. The car will then drive into the herd of cows.
4. If the magnitude of μ_d mg cos θ l is large enough, then K2 becomes negative.	F	4. Kinetic energy can never be negative. The car will stop before it reaches 2.

5.3.4. P

A car with mass m has driven up a hill and then down a slope with inclination θ. As the driver passes point 2, he sees a herd of cows blocking the road further down at point 3 He slams on the brakes, and the car starts to skid over the wet surface of the road. The distance over the surface of the road from 1 via 2 to 3 is ℓ. The coefficient of dynamic friction between the tyres and the road surface is μ_k. The car experiences an air resistance, which is proportional to the speed squared.

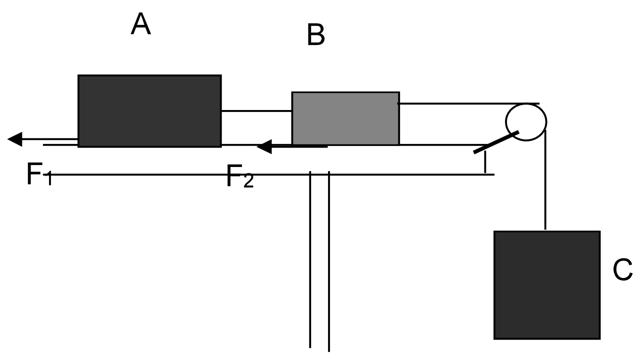


Test Statement	T/F	Because
1. It is possible to introduce a potential energy function for the force acting on the tyres of the car while it is skidding down from 2 to 3.	F	1. This force is a friction force, and thus non-conservative.
2. There is an increase in gravitational potential energy of the car as it moves from 1 to 2.	T	2. When the car climbs, its gravitational potential energy increases.
3. Not to drive into the cows, the driver must make the gravitational potential energy at 3 zero.	F	3. The kinetic energy must be zero.
4. The loss of gravitational potential energy as the car moves from 1 to 3 is mg ℓ sin θ.	F	4. The difference in height between 1 and 3 is smaller than ℓ sin θ.

5.4 Appendix: Additional Test Units for Friction: A - E

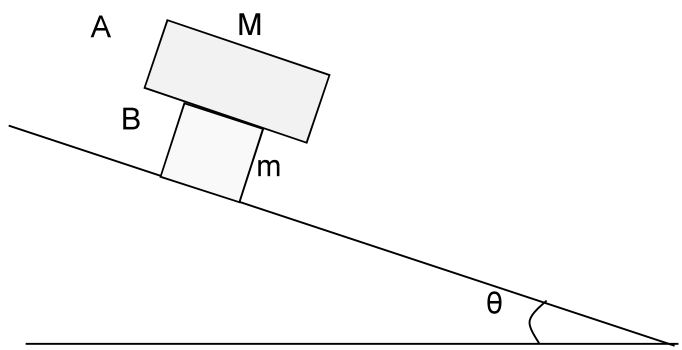
5.4.1.**The influence of frictional forces on the motion of a system**

5.4.1.A Two blocks, A and B lie on a horizontal table. A and B are connected by a string, and another string connects B to block C via a frictionless pulley. C is hanging freely from this string. The coefficient of dynamic friction between the table surface and blocks A and B (see figure) is $\mu_d = 0.2$. The mass of block A is *2m*, the mass of block B is *m*, and that of block C is *3m.*



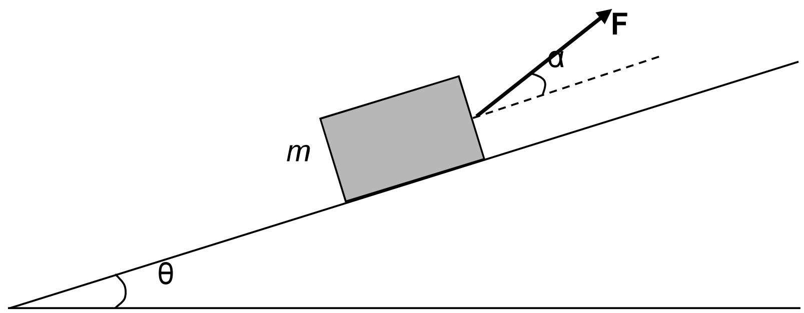
Test Statement	T/F	Because
1. A and B are moving to the right with constant acceleration	T	1. There is a constant net force acting on A and B, giving this system a constant acceleration.
2. The tension in the string supporting C is *3mg*	F	2. C has an acceleration downwards so this tension is < *3mg*
3. The tension in the horizontal part of the string connecting B with C equals the tension in the vertical part.	T	3. The pulley only changes the direction of the force in the string and does not change its magnitude.
4. The tension in the string connecting A to B equals the tension in the string connecting B to C.	F	4. The tension between A and B is smaller than the tension between B and C.
5. If the mass of C were to be doubled to *6m*, then the acceleration would also double	F	5. The acceleration depends on both the mass and the net force on the system, and does not double.

5.4.1.B. Two wooden blocks, A and B, with mass M and m respectively, rest on an inclined plane. The coefficient of static friction between block A and block B is μ_1 and that between B and the surface of the plane μ_2. $\mu_1 > \mu_2$ and $M > m$. The angle of the slope θ is small enough to have the whole system at rest.



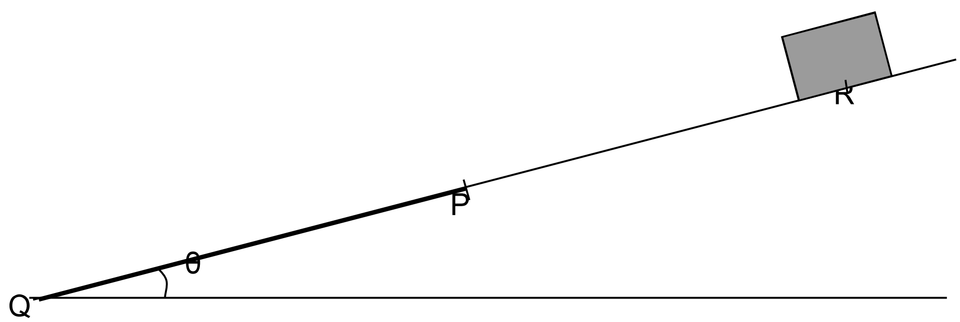
Test Statement	T/F	Because
1. If you pull on B with a force that is just large enough to make B slide upwards with constant velocity, then A starts sliding relative to B	F	1. A remains in the same position relative to B because $\mu_1 > \mu_2$
2. If you pull on B upwards with a large force, then A slides off B	T	2. The friction between the blocks will not be large enough to give A the same acceleration as B gets.
3. If θ is slowly increased, then A, having a larger mass, will start sliding over B before B starts sliding over the plane.	F	3. B will start sliding down before A starts sliding over B
4. It is possible to find a value of θ such that once set moving, A and B move together down the slope with a constant velocity.	T	4. The slope that makes B slide with constant velocity over the plane is not large enough to set A sliding over B.

5.4.1.C. A polished wooden block with mass *m* is resting on an inclined plane also made from polished wood. The coefficient of dynamic friction between the block and the plane is μ, and the angle of the slope is θ. A force **F** is applied to the block. The angle between **F** and the slope is α. $\mathbf{F_1}$ is a special case of **F**, that makes the block move up the slope with constant velocity.



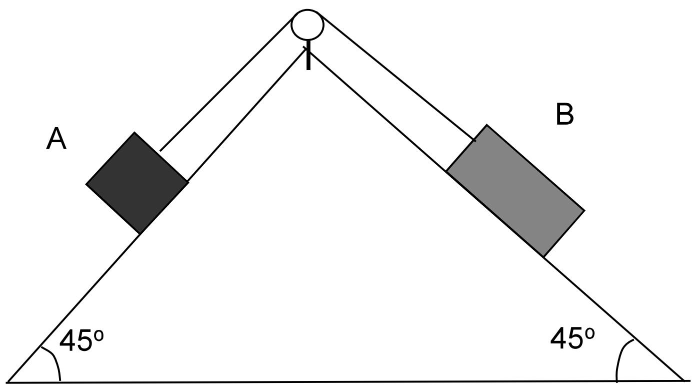
Test Statement	T/F	Because
1. If $\alpha = 0$, and θ is kept constant, while *m* and μ vary, then the magnitude of $\mathbf{F_1}$ is proportional to *m* and μ.	T	1. The magnitude of $\mathbf{F_1}$ is proportional to both *m* and μ.
2. If $\alpha = 0$, and *m* and μ are kept constant, while θ varies, then the magnitude of $\mathbf{F_1}$ is proportional to $\cos\theta$.	F	2. The magnitude of $\mathbf{F_1}$ is a function of both $\cos\theta$ and $\sin\theta$.
3. If $\alpha > 0$, then the force of friction along the plane is smaller than if $\alpha = 0$.	T	3. The normal force on the block is smaller, and so is the force of friction.
4. If, with $\alpha = 0$, the force $\mathbf{F_1}$ makes the block move upwards with constant velocity, then a force $\mathbf{F_2} = -\mathbf{F_1}$ will make the block move downwards with constant velocity.	F	4. In both cases a component of gravity acts downwards, so the magnitude of $\mathbf{F_2}$ is smaller.

5.4.1.D. A block is moving on an inclined plane that has two areas: from R to P it is frictionless, and from P to Q there is a coefficient of dynamic friction μ between the block and the surface. The angle of the slope is θ. The mass of the block is m. The block is at rest at R and is let free to move with initial velocity zero.



Test Statement	T/F	Because
1. When the block reaches P it suddenly stops.	F	1. That would require infinite force acting on the block at P.
2. At P the motion of the block changes from motion with constant acceleration to motion with constant velocity.	F	2. Below P the block still moves with (a different) constant acceleration.
3. If θ is small enough, the block will slow down and come to rest between P and Q.	T	3. The friction force is then large enough to stop the block.
4. If the block is set to move up the plane from Q, and has enough velocity to pass P, then it will go on moving with constant velocity above P.	F	4. There is no friction above P, but gravity has a component along the plane, which will slow down the block.

5.4.1.E. Two blocks A and B with masses *m* and *2m* respectively lie on two inclined planes. A and B are connected by a massless string via a frictionless massless pulley as shown in the figure. Both planes have a slope of 45^o. The coefficient of dynamic friction between the planes and each of the blocks are both μ. Initially the whole system is at rest.



Test Statement	T/F	Because
1. If μ is small, then B moves down the slope.	T	1. B will move downwards.
2. If μ is very large, then A and B remain at rest.	T	2. The force of friction on B is then large enough to compensate the component of gravity along the slope.
3. It is possible to find a value for μ such that B moves down the slope with constant velocity.	T	3. There is one value for μ that makes this motion possible.
4. It is possible to find a value for μ such that A moves down the slope with constant velocity.	F	4. In no circumstances can A move down pulling B upwards.