

Chapter 6

Percentages and Diagrams

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Introduction

This chapter was made to replace chapter 6; Percentages and Diagrams of Numbers and Space for havo tto 2 classes.

This booklet contains ten lessons (two pages per lesson) which all consist of:

- Theory** In this part of the lesson, new mathematics is explained.
- Example** An example about the new theory is worked out. The calculations are given clearly. You can use this example to check how to apply the things you just learned.
- Homework** These are the exercises you do to practice working with the theory.
- Checklist** Use the checklist to check if you understand all the important learning goals of the lesson.

In lesson 7 and 8 you will work on a project.

After the ten lessons, you can find the star exercises. These are exercises that require a bit more insight and skill than the homework exercises.

After the star exercises you can find the Diagnostic Test. This is practice material for your final test.

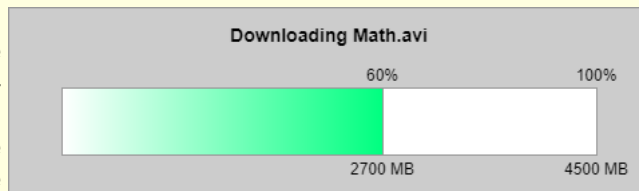
At the end of the booklet, you can find the Overview of Terms. This is a list of all important words you should be able to understand and use in both Dutch and English. If you do not know a mathematical term in a lesson, you can find it in this list.

Lesson 1: Prior knowledge: Calculating with Percentages

Theory

The green download bar shows how a downloading process could look like on your phone. Sometimes only the percentage is given above the bar, or the amount downloaded is given below the bar.

If the percentage is given we can find the amount of MB already downloaded by doing the following:



The whole bar is 100% → 4500 MB

: 100

1% is then $\frac{1}{100} \times 4500$ MB

× 60

60% is then $60 \times \frac{1}{100} \times 4500$ MB

This means you have downloaded $\frac{60}{100}$ part of 4500 MB. We can calculate $\frac{60}{100} = 0.6$. Using this you can calculate the amount of MB you have already downloaded:

60% of 4500 MB is $0.6 \times 4500 = 2700$ MB

Sometimes only the amount of MB you have already downloaded is given instead of a percentage. Then we can find the percentage by calculating $\frac{2700}{4500} \times 100\% = 60\%$.

If we look at the blue download bar we see that we only have an amount given, but we can calculate the percentage.

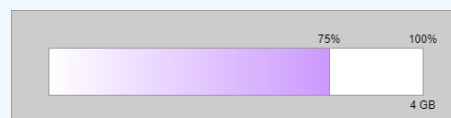
1250 MB of 4500 MB is $\frac{1250}{4500} \times 100\% \approx 22.8\%$

You should always round a percentage to one decimal place.



Example

Mike is downloading a movie of 4 GB and the download is at 75%. How much GB has already been downloaded?



If we split up the bar into 4 pieces, each piece will be 25% and will be 1 GB. So 75% will be 3 GB. We can check this with a calculation: $0.75 \times 4 = 3$ GB. So Mike has already downloaded 3 GB.

Homework

1. First draw a download bar and then calculate:

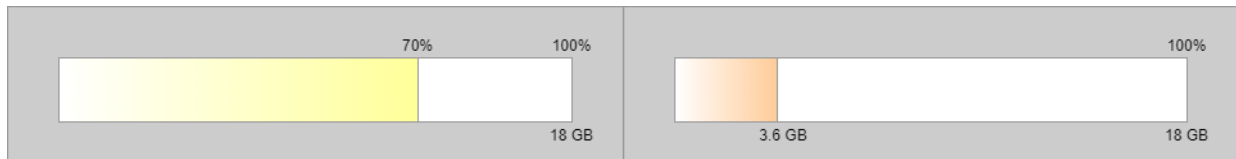
- (a) 10% of 260
(b) 25% of 80

- (c) 33.3% of 2100
(d) 1.5 out of 2

- (e) 850 out of 4250
(f) 180 out of 185

Check if your answer corresponds to your drawing.

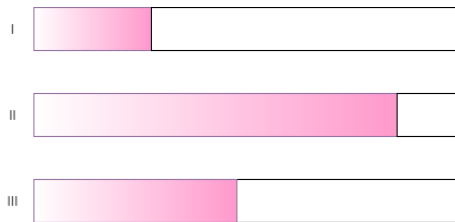
2. a. Yasmina is downloading a game from Steam. The game is 18 GB big. Yasmina sees the download bar in figure a. How much GB does she still need to download?
b. Eric is also downloading this game but on another platform. He sees the download bar in figure b. What percentage has already been downloaded?



(a) Yasmina

(b) Eric

3. a. What percentage belongs to which bar?
b. If the bars represent 5 GB each, how much GB has been downloaded for each bar?

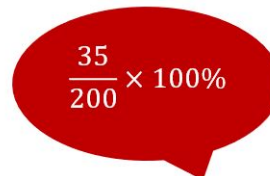


- A. 85%
B. 28%
C. 46%

4. Giovana is downloading a song. The size of the song is 3.8 MB. The download is at 47%. Draw the download bar and try to estimate how much MB has already been downloaded. Then calculate it exactly.
5. Achmed and Lucy are discussing how to calculate how much 35% of 200 GB is. Who is right and why? What mistake is the other person making?



Achmed



Lucy

Checklist

- Can you name an example where we use percentages in the real world?
- If you are downloading a 2 GB movie at home and it is at 1 GB, at what percentage is it?
- If you are downloading a 4 GB movie at home and it is at 25%, how much GB has been downloaded?

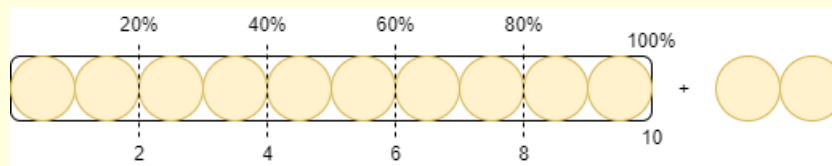
Lesson 2: Increase with a Percentage

Theory

Percentages are used to show an increase. For example, the price of a certain pair of shoes has been increased by 8%. Sometimes packages of sweets also are increased by percentages. For example we can look at a candy roll. Originally the candy roll contained ten candies:



Now the number of candies will be increased by 20%. When we split up the bar in pieces of 20% we have 5 parts.



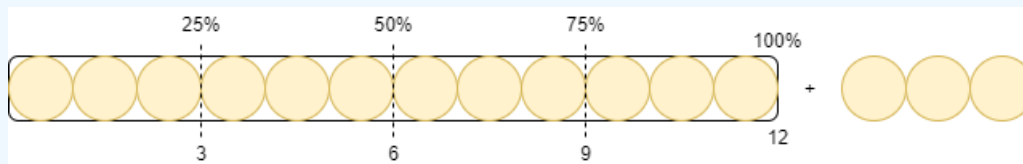
Each part is 2 candies, accordingly an increase with 20% will increase the bar with 2 candies. So when we add these two candies to the old roll we have 12 candies. We also have a formula to calculate this:

$$\begin{aligned} \text{new amount} &= \text{old amount} + 0.20 \times \text{old amount} \\ \text{new amount} &= 10 + 0.20 \times 10 = 10 + 2 = 12 \end{aligned}$$

The 0.2 is again the percentage divided by 100, so $\frac{20}{100} = 0.2$.

Example

A candy roll contains 12 candies. A new packaging will increase the number of candies by 25%. How many candies will the roll contain now?



We first want to find out how many candies correspond to 25%. Because 25% fits four times into 100%, we split the candy roll into 4 equal parts. Each part contains 3 candies. So, when we increase with 25% we have to add 3 candies, which gives a total of 15 candies:

$$\text{new} = 12 + 0.25 \times 12 = 12 + 3 = 15$$

Homework

1. A candy roll has 20 candies in it. A new package increases it with 15%. How many candies are in the new roll? Show in the bar and calculate.



2. Calculate the new amount using your calculator. Round to one decimal place. Draw a percentage bar for questions a, b and c.
- | | | |
|----------------------------|----------------------------|-----------------------------|
| (a) 20 increased with 5% | (c) 350 increased with 40% | (e) 823 increased with 2.5% |
| (b) 100 increased with 40% | (d) 59 increased with 75% | (f) 32.5 increased with 9% |
3. In 2021, there are 1500 students who take Dutch education and 405 students who learn in English and Dutch at school (tto). In 2022, there are 3% more students for the Dutch program.
- How many students follow the Dutch program in 2022?
 - In 2022 there are 11 more students following the tto program than in 2021, so now there are 416. What percentage of the 416 are new?
4. It is well-known that too much sugar is bad for you, and that there is a lot of sugar in our food and in soft drinks. To prevent people from buying too much soft drinks, the United Kingdom introduced a sugar tax. The price for one can of Coca-Cola was 0.55 pound and is now increased by 20%. How much do you have to pay for a can of Coca-Cola now?
5. Joseph wants to buy a new bike. There are two shops in town that sell bikes. Tires&Brakes sold bikes for 650 euros, but has increased the price by 20%. Bikes&Co sold bikes for 720 euros, but has increased the price by 10%. Joseph wants to buy the bike from Bikes&Co, but Ali thinks that he would be better off with a bike from Tires&Brakes. Who is right and why?

Checklist

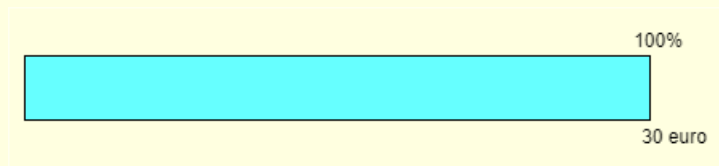
- Your favourite game costs 20 euros. Now the price has increased with 10%, can you calculate how much it is now?

Lesson 3: Decrease with a Percentage

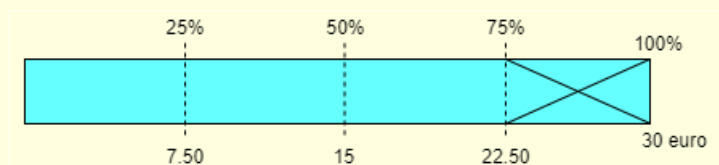
Theory

We can also use percentages to talk about a decrease, so something becoming less with a certain percentage. For example last years sweaters are being sold with a 25% discount. So the price is decreased with 25%.

For example the sweater was originally priced 30 euros. We will look at this price as a bar:



25% fits exactly four times in 100%, so we split the bar into four equal parts.



Each part is 7.50 euro, so the sweater will now cost $30 - 7.50 = 22.50$. We can also calculate this with this formula:

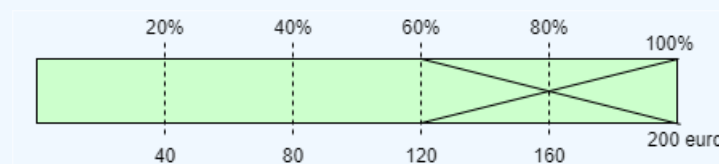
$$\begin{aligned} \text{new amount} &= \text{old amount} - 0.40 \times \text{old amount} \\ \text{new amount} &= 30 - 0.25 \times 30 = 30 - 7.50 = 22.50 \end{aligned}$$

This formula looks a lot like the one in lesson 2. But notice the $-$ instead of the $+$. For increase we use the $+$, for decrease we use the $-$.

Example

A new mobile phone costs 200 euros. You have a coupon for 40% discount. How much does the phone cost now?

We cannot split up 100% in equal parts of 40%. But we can split 100% in 5 parts of 20%, and then take two.

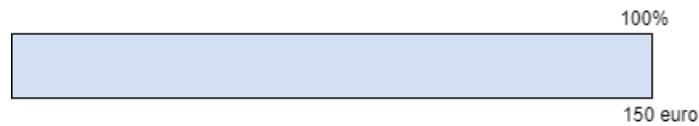


We see that 40% is 80 euros. So the phone will be 120 euros. We can calculate this:

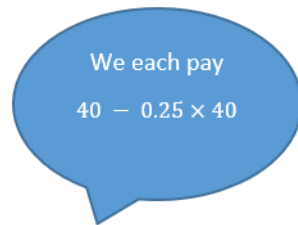
$$\text{new amount} = 200 - 0.4 \times 200 = 200 - 80 = 120.$$

Homework

1. Lenne wants to buy shoes worth 150 euros. The shoes are in the sale with a discount of 33%. How much are the shoes now? Estimate the answer with a bar and then calculate.



2. Calculate the new amount, round to one decimal place:
- (a) 40 increased by 8% (c) 600 decreased by 40% (e) 645 decreased by 2.5%
(b) 100 decreases by 17% (d) 59 increased by 24% (f) 86.3 increased by 16%
3. a. A bag of crisps used to weigh 200 grams. The factory has decreased it by 40%. How much does it weigh now?
b. A membership to an online streaming platform costs 7.50 euros per month. You have a code for a discount of 5% for one month, how much will that month cost?
4. a. Ethem spends 4.5 hours gaming on Saturday. On Sunday he spends 23% less. How many hours and minutes does he spend on the computer on Sunday? Round to whole minutes.
b. Ines spends 3 hours of Sunday on her mathematics homework. What percentage of the day did she spend on her homework?
c. Damian spends an hour watching television on Friday. On Saturday he watched 120% more television. How many hours and minutes did he spend watching television on Saturday?
5. Stan and Misha both want to buy a game. The game costs 40 euros, but if they buy two games at once, both games get a friend discount of 25%. They have the ideas shown below. Who is right? Why? Explain.



Stan



Misha

Checklist

- Your favourite chocolate is 3 euros, now the price has increased with 33%, can you calculate how much it is now?
- A sweater costs 25 euros, but it is on sale with a discount of 20%, can you calculate how much it costs now?

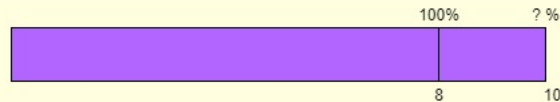
Lesson 4: Calculating percentage increase and decrease

Theory

Suppose that your salary increases from 8 euros per hour to 10 euros per hour. You want to know with what percentage your salary has increased. To calculate this, you first need to calculate the **absolute** increase. This is the amount of euros your salary increases. In this case,

$$\text{absolute increase} = \text{new amount} - \text{old amount} = 10 - 8 = 2 \text{ euros}$$

The **relative** increase is the increase in percentages. This can be illustrated with a percentage bar:



The relative increase is the percentage with which the bar has increased. Here the bar is increased with 25% since 2 out of 8 is 25%. We can also use a formula to calculate this:

$$\text{percentage increase} = \frac{\text{increase}}{\text{old amount}} \times 100\% = \frac{10-8}{8} \times 100\% = 25\%$$

Now suppose that your salary decreases from 8 euros per hour to 6.50 euros per hour. The percentage decrease can be calculated as follows:

$$\text{percentage decrease} = \frac{\text{decrease}}{\text{old amount}} \times 100\% = \frac{8-6.50}{8} \times 100\% \approx 18.8\%$$

We don't always write the word 'absolute'

Round percentages to one decimal place

Note that in both cases you calculate the difference in percent with $\frac{\text{difference}}{\text{old amount}} \times 100\%$.

Example

Justin is 164 cm tall. He needs to be 170 cm tall to join the basketball team. It is unlikely to grow more than 4% per year.

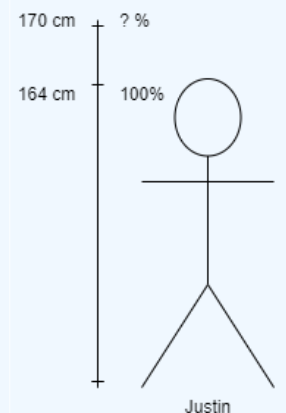
- a. **What percentage does Justin need to grow to join the basketball team?**

Increase = 6 cm and old amount = 164 cm.

$$\text{So percentage increase} = \frac{6}{164} \times 100\% \approx 3.7\%$$

- b. **Is it plausible that Justin joins the basketball team in one year or is that unlikely?**

3.7% is almost 4%, but it is still less than 4%. So it is plausible that he joins the basketball team in one year.



Homework

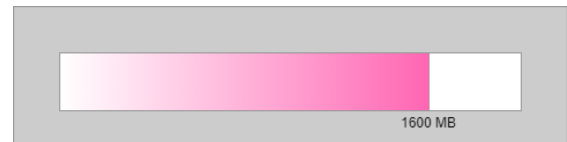
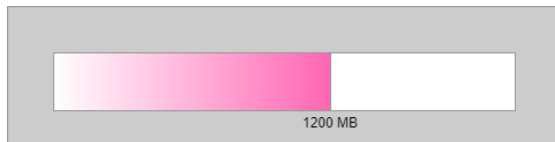
Table 1

1. Relative increase
2. Relative decrease
3. Absolute increase
4. Absolute decrease

Table 2

A. Increase of 20 kg
B. Increase of 12%
C. Decrease of 40 cm
D. Decrease of 3%

1. Connect each term from Table 1 to a term from Table 2.
2. Look at the example.
 - a. People are less tall in the evening than in the morning. Justin did measure 164 cm in the morning. In the evening, he is 162 cm tall. What is the relative decrease of his length during a day?
 - b. What percentage does Justin need to grow to join the basketball team if he uses his evening length?
 - c. Is it still plausible that he joins the basketball team in one year or has it become unlikely?
 - d. Would you already be able to join the basketball team? If not, what percentage do you still need to grow? If yes, calculate the percentage increase from 170 cm to your length.
 - e. Is it plausible that you could join the basketball team in one year or is that unlikely?
3. Amber is downloading a movie of 2000 MB. She has already downloaded 1200 MB. A few minutes later, the download is at 1600 MB. The download bars for both situations can be seen below.



- a. Calculate at what percentage the download is in both situations.
 - b. What is the percentage increase in MB?
 - c. Is your answer from part (b) the same as the difference in percent between the two download bars? Why (not)?
4. Karim is selling roses for 1 euro per rose. To get more money, Karim increases the price to 1.20 euro per rose.
 - a. Show that the relative increase is 20%.
 - b. People think this is too much money and they do not buy the roses anymore. So Karim lowers the price of 1.20 euro by 20%. Without calculating, explain if the roses will cost less than 1 euro, if they cost 1 euro or if they cost more than 1 euro.
 - c. Calculate the new price and explain your observation. Was your idea from part (b) correct?

Checklist

- Can you calculate the absolute increase if your salary goes from 7 to 9 euros per hour?
- Can you calculate the relative increase if your salary goes from 7 to 9 euros per hour?
- Can you calculate the absolute decrease if your salary goes from 14 to 11 euros per hour?
- Can you calculate the relative decrease if your salary goes from 14 to 11 euros per hour?

Lesson 5: Bar and Line charts

Theory

Data is everywhere around us. It can be a list of numbers or a list of words. We call these the data values. But it is really hard to learn something from a list. So we are going to look at ways to visualise data, to make a picture of the data.

One of the ways to visualise is to create a **bar chart**.

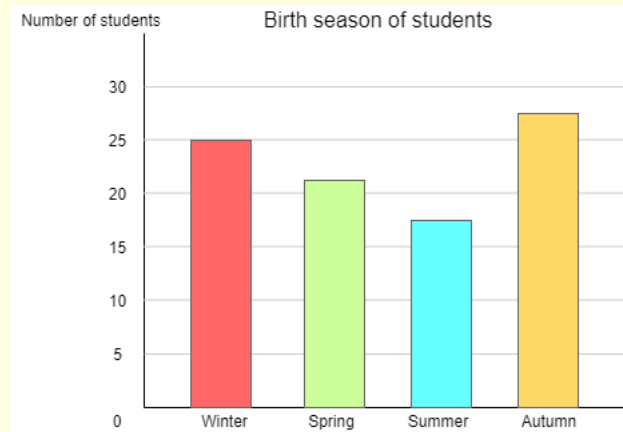
Rules for drawing a bar chart

- A bar chart has a title
- Label the axes
- The bars don't touch each other
- The size of the bar corresponds to the amount it represents

The dates on which seasons start can vary each year, but most people use following general dates for the seasons:

- Winter: December 21 - March 20
- Spring: March 21 - June 20
- Summer: June 21 - September 20
- Autumn: September 21 - December 20

Figure 2



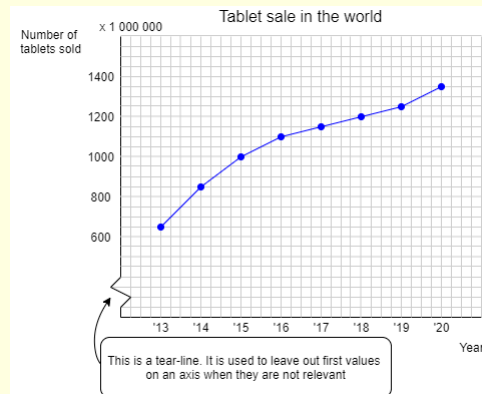
The bars can be both vertical (as in figure 2) or horizontal. The length of the bar shows in this example how many students were born in the winter (25), spring (21), summer (17) and autumn (27).

To visualise data, we can also use a **line chart**, as in figure 3. Here the tablet sale in the world between 2013 and 2020 is represented in a line chart.

Rules for drawing a line chart

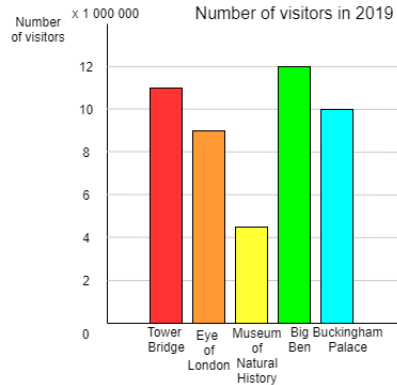
- A line chart has a title
- Label the axes
- Place a dot for each data point and connect those dots with a line
- Use a tear line if needed

Figure 3



Homework

1. In the bar chart the number of visitors in 2019 of some popular attractions in London can be seen.
- a. Estimate for each attraction how many visitors visited this in 2019.



- b. The number of visitors for Buckingham Palace in 2018 was the same as the number of visitors for the Big Ben in 2019. What is the percentage decrease of the number of visitors of Buckingham Palace from 2018 to 2019?
- c. In 2019, 2 million students visited the Museum of Natural History. What percentage of the visitors of the Museum of Natural History were not students in 2019?

2. The average temperature in degrees Celsius per month in Sevilla, a city in Spain, in the first six months of a year is given in the table below:

Month	Temperature in degrees Celsius
January	16
February	17
March	20
April	22
May	27
June	32

- a. Draw a line chart for this data. Let 1 cm represent 2 degrees Celsius on the vertical axis and use a tear line.
- b. What is the percentage increase in degrees Celsius from January to April?
- c. Is it also possible to visualise this data with a bar chart? Why (not)?
3. Students are discussing the line chart in figure 3. They say the following:
Jasper: "The amount of tablets sold in 2018 is about 5 times as high as in 2013."
Li-Ming: "The amount of tablets sold in August 2013 are higher than in February 2013."
Nina: "The amount of tablets that are sold worldwide between 2013 and 2020 has increased."
 Discuss these statements with another student.

4. There are 25 students in a 2nd class. One student has asked each student for their age and tallied it. Create a bar chart from this information.

Age	Tally
12	
13	
14	
15	

Checklist

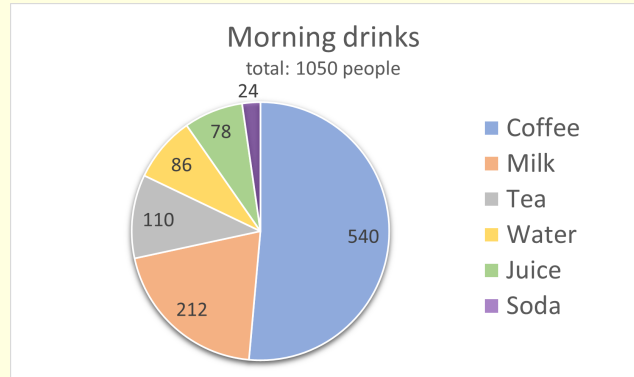
- Could you create a bar chart containing the ages of your classmates following all the rules?
- Could you create a line chart from a table following all the rules?
- Can you read information from a line or bar chart?
- Can you use the information found to calculate differences in percentages?

Lesson 6: Pie Chart

Theory

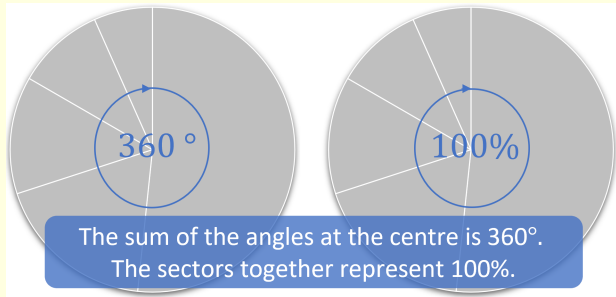
A newspaper presents the following pie chart about morning drinks. They claim that more than 54 % of the people drink coffee in the morning. A pie chart is a diagram in the shape of a circle. It is used to easily visualise data. The circle is divided into sectors. The size of each sector indicates the percentage of the whole.

You can use what you learned about percentages to calculate the central angles of the sections as follows: **central angle** = $\frac{\text{percentage}}{100\%} \times 360^\circ$.
 You can also use amounts for this:
central angle = $\frac{\text{amount}}{\text{total amount}} \times 360^\circ$



How to draw a pie chart:

1. Calculate the central angles of each sector
2. Draw a circle and a vertical radius
3. Draw the central angles clockwise and colour the sections
4. Write a title and a legend



Example

The data used in the pie chart is shown in the table.

Pet:	Dog	Cat	Bird	Fish	Others	Total
Percentage:	51.7%	18.3%	13.3%	10%	6.7%	100%

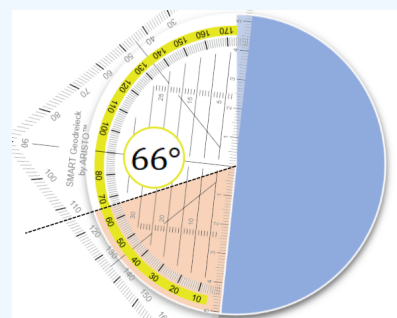
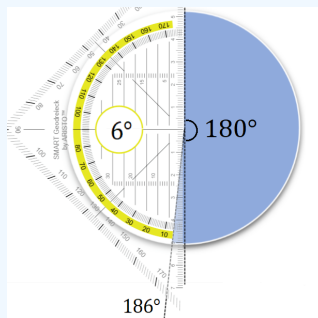
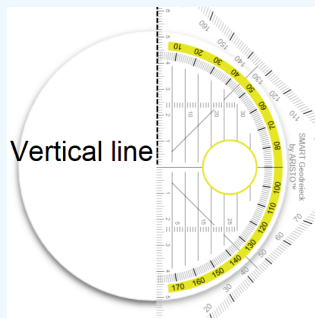
Dog: $\frac{51.7\%}{100\%} \times 360^\circ \approx 186^\circ$

Cat: $\frac{18.3\%}{100\%} \times 360^\circ \approx 66^\circ$

Bird: $\frac{13.3\%}{100\%} \times 360^\circ \approx 48^\circ$

Fish: $\frac{10\%}{100\%} \times 360^\circ \approx 36^\circ$

Others: $\frac{6.7\%}{100\%} \times 360^\circ \approx 24^\circ$

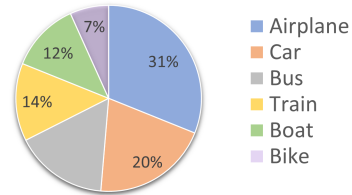


Homework

- Complete the pie chart from the example.
- A group of people were questioned about which transport method they used for the holiday of 2019. The result of the survey can be found in the pie chart.

- How many people went on holiday by train?
- 12 people went on holiday by bus (the grey area). Calculate the corresponding percentage.
- How big is the angle you have to draw for the sector 'airplane'? Round to whole degrees.

Which transport did you use at the holiday of 2019?
total: 74 people

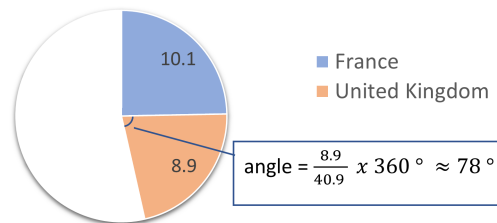


- The table and the pie chart show how many films and series per country have been downloaded via Netflix. The total of the six countries is 40.9 million downloads.

Country	France	United Kingdom	Germany	Spain	Italy	The Netherlands
Downloads (in millions)	10.1	8.9	...	5.6	4.9	2.7

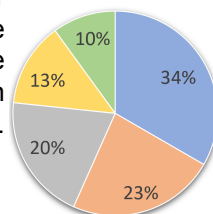
Number of downloads on Netflix
total: 40.9 million

- How many downloads are there in Germany?
- Complete the pie chart.
- What percentage of downloads are there in France?

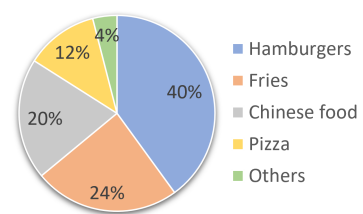


- In two classes, students were asked what their favourite food was. The results are shown in the two pie charts. Andrew claims that more people from group A like Chinese Food than people in group B like fries? Oliver claims the opposite. Who is right? Explain!

Favourite food class A
total: 30 people



Favourite food class B
total: 25 people



- Simon has written down what his day looks like. Draw a corresponding pie chart.

Activity	Eating	School	Homework	Gaming	Sleeping
Hours	1	7	4	3	9

Checklist

- Can you name an example where we use a pie chart in the real world?
- Can you calculate the central angle of a sector if 6 people out of 12 gave the same answer?
- Can you calculate the central angle of a sector if the percentage for that sector is 25 percent of the total?
- Can you calculate how many people gave the same answer out of 50 people in total, if the sector for that question has a central angle of 72 degrees?

Lesson 7 & 8

Goal

More and more teenagers and young adults overwork themselves and get burn-outs. It is important to spend enough time on recreation so the chance of getting a burn-out is lower. We want to find out how often you spend time on hobbies and how much time you need to go to school. In this project, you are going to visualise data to get an better overview of these aspects. You will receive two data sets (data set A: hours spent on hobbies per week and data set B: Time spent on commute by bike or other) and analyse these, so we can compare and visualise data. Although you work in groups, you must write down all the solutions individually, so you can explain and show the answers to another student that worked with a different data set.

Execution

Look at the data set A. Each number represents how the hours per week spent on hobbies by one of your classmates. We want to organise the data, so we can see how many times a certain number occurs. How often a value occurs is called the **frequency** of a value.

1. Fill in the frequencies in table 3.

Table 3: Frequency table for hobbies

Hours spent on hobbies per week	≤ 4	5	6	7	8	9	≥ 10
Frequency							

Theory

This table is called a **frequency table**. A frequency table consists of a column with the data values and how many times this value occurs. These numbers are called the **frequencies**.

Example

We asked students about how many hours they game on a school day. We found the following results:
2 5 1 0 1 5 3 3 6 2 2 0 2 4 8 4 2 3 2 2 2 1.
The corresponding frequency table is:

Table 4: Frequency table for gaming

Hours of gaming on school day	0	1	2	3	4	5	6	7	8
Frequency	2	3	8	3	4	2	1	0	1

In this table you can see that 8 students game 2 hours on a school day by looking at the frequency under the number 2.

Frequency tables give a nice overview of the data set, but now we want to see the data in a picture.

2. Make a bar chart from this frequency table, but draw the bars without gaps between them.

Theory

A bar chart for a frequency table is called a **histogram**. There are no gaps between the bars.

3. Discuss and compare your frequency tables and histograms in your group. What relation between the frequency table and histogram can you see?

For the next exercises, we take a look at data set B, which shows the time you need for your commute (your trip from home to school). To get an overview of the travelling times of you and your classmates, you can make a **stem-and-leaf diagram**.

Theory

A **stem-and-leaf diagram** is a table where all data points are ordered. The numbers are split in two parts: the amount of tens and the amount of units in the number. The tens are put in the stem of the diagram, the ones are put in the leaves. A **double stem-and-leaf diagram** has leaves on either side of the stem. The information in the leaves are split into two groups, one group will be put on the left and one on the right.

Example

We also asked students to keep track of the number of hours they sleep in a week. We put the data in a stem-and-leaf diagram, see diagram 5. In this diagram you can see that there were 3 students who slept 53 hours in a week. You look at the tens for 5, and then look at the 3 in the units. Because this number occurs 3 times, the frequency of the data value 53 is 3.

Table 5: Stem-and-leaf diagram Hours of sleep in a week

Tens	Units
3	9
4	2 4 6 8 9 9
5	0 1 3 3 3 4 4 5 6 6 6 8
6	3 6 8 8
7	1

Table 6: Double stem-and-leaf diagram for travelling times

Commute by bike		Commute by other forms of transportation
	0	
	1	
	2	
	3	
	4	
units	tens	units

4. Fill in table 6 with the data of set B. The stem is already given. Put the data in the leaves in order from smallest to largest. Make sure you put the data on the correct side of the stem.
5. Why is it not a good idea to make a frequency table for this data?

Theory

In a stem-and-leaf diagram, the data is nicely ordered into groups. We can make a **frequency distribution** for this data. These groups are called **class intervals**. The lowest and highest number that can be put into a class interval are the **class boundaries**. Note that in the notation 30-<40, 30 is included in the class interval, but 40 is not. The **class width** is the number of values that can be put into a class.

Example

We use the data set about the number of hours that students sleep (from the example on page 14) in a week again to make a frequency distribution. The stem-and-leaf diagram makes it easy to let the class width be 10. We get the following table:

Table 7: Frequency distribution for hours of sleep

Class interval	Frequency
30 -< 40	1
40 -< 50	6
50 -< 60	12
60 -< 70	4
≥ 70	1

In table 7 we see that the class width is indeed 10 and the class boundaries are the multiples of 10. Note that the last class interval in this table consists of all numbers larger or equal to 70.

Look at the theory on page 14 about the class intervals again.

6. What would be logical class intervals to order the data of set B in?
7. What are the class boundaries of your groups?
8. Draw up a frequency distribution of the travelling times with interval width 10 (use your stem-and-leaf diagram) like the one in table 7 in the example.

Discussion

We asked students how many hours they spent on different activities during a day. We made two pie charts, one for the week days and one for days in the weekend. Look at figures 5 and 6.

Figure 5: week days

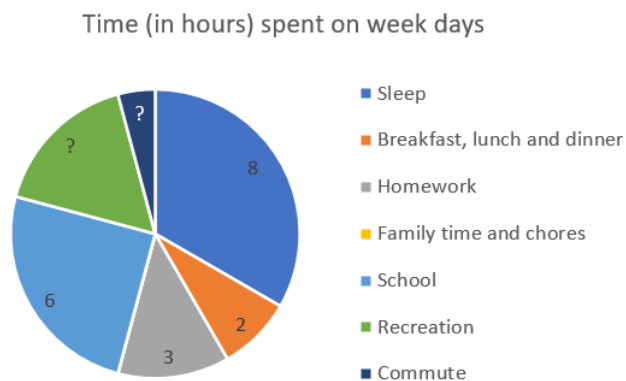
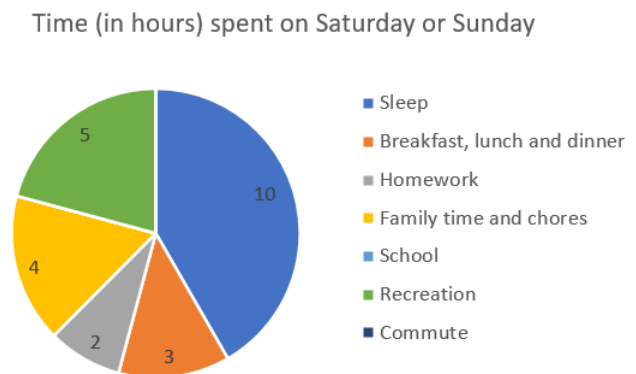


Figure 6: Saturday or Sunday



In figure 5 we see that there are 5 hours for recreation and commuting combined (24 hours in a day, $24 - 8 - 2 - 3 - 6 = 5$, so 5 hours left). In figure 6 we see that there are 5 hours for recreation on both Saturday and Sunday. Thus, students spent $7 \times 5 = 35$ hours, minus 10 times the commute time (5 commutes to school and 5 commutes back) on recreation.

Discuss the following questions:

1. Are there differences in your stem-and-leaf diagrams? What could explain those differences?

2. Are there differences in your frequency distributions? What could explain those differences? Look at the commute times of the students in both data sets. What is the longest commute time? And the shortest? What can you say about the leftover recreational time per week for those students?
3. What did you use to answer the above question? The frequency table or the histogram? Why?
4. What are differences in your frequency tables?
5. What are differences in your histograms?
6. Look at the data on hobbies in both data sets. What is the largest number of hours spent on hobbies? And the smallest number of hours?
7. Suppose the student with the longest commute time is also the student with the most hours spent on hobbies. Is this possible? If so, how much recreational time does this student have left in a week?
8. Suppose the student with the shortest commute time is also the student with the least hours spent on hobbies. How much recreational time does this student have left in a week?
9. Compare these two students. What do you think about the recreational time the students have left? Is there a chance for a burn out?

Checklist

- Can you make a frequency table and draw the corresponding histogram for a data set?
- Can you explain the data represented in a histogram?
- Can you explain the data represented a stem-and-leaf diagram?
- Can you make a frequency distribution and write down the class boundaries and class width?

Lesson 9: Median, Mode and Mean

Three students, Sophie, Aisha and Peter want to compare the pocket money of a group of 9 boys and a group of 8 girls. They each have their own idea about how to compare this. They ask each of the boys and the girls to tell them the amount of pocket money they get every week.

Sophie's idea
We order the amounts of pocket money from low to high for the boys and the girls. Then we compare the middle number of the boys with the middle number of the girls.

Aisha's idea
We compare the amounts of pocket money that is given most often mentioned by the girls with the one that most often mentioned by the boys.

Peter's idea
We take all the pocket money of the boys, add it up and share it among the boys. We do the same for the girls and we compare these shares.

Which idea is best suited to compare the pocket money of the two groups?

Theory

The median, mode and mean are three ways to describe a central position in a set of data. In the case above the set of data is the amounts of pocket money mentioned by the groups. These amounts are called data values.

<p>Median (Sophie) Order the set of numbers and choose the middle one. 9 3 1 8 3 6 1 3 3 6 8 9 if there is no middle number we can calculate it: $\frac{3+6}{2} = 4.5$. So the median is 4.5</p>	<p>Mode (Aisha) The most frequent number. 9 3 1 8 3 6 So the mode is 3.</p>	<p>Mean (Peter) Add all the numbers and divide by the amount of numbers. $\frac{9+3+1+8+3+6}{6} = \frac{30}{6} = 6$ So the mean is 6.</p>
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These three terms, median, mode and mean, are called '**central tendencies**'.

Example

The pocket money of the girls is:
3.50 4 7 6 7.50 10 5 4.
Calculate the median, mode and mean. Round to one decimal place.

Median: Ordering the values gives 3.50 4 4 5 6 7 7.50 10. So the median is $\frac{5+6}{2} = 5.5$.

Mode: 3.50 4 7 6 7.50 10 5 4, so the mode is 4.

Mean: $\frac{3.50+4+7+6+7.50+10+5+4}{8} \approx 5.9$

Homework

1. The pocket money of the boys are:
10 3 6 2.50 3 6 9 6 5.50.
Calculate the median, mode and mean for the boys and compare them to the girls in the example.
Round to one decimal place if necessary.

2. In a factory are working 10 different staff-members. The salary of each staff member is shown below:

Staff	1	2	3	4	5	6	7	8	9	10
Salary in euros ($\times 1000$)	15	18	16	14	15	15	12	17	90	95

- What is the mean salary for these ten staff members?
 - Lindsey is inspecting the data and she thinks that the mean might not be the best option to say something about the typical salary of a staff-member in this factory. Why does she think this?
 - Brenda agrees with Lindsey and she would like to have a better measure of central tendency. Which measure do you suggest to Brenda and Lindsey and why?
 - Calculate the median and the mode of the salaries of these ten staff members.
3. Alex is a class tutor and he wants to organise a meeting with the parents of his class. Because he wants to give the parents a present he asks the students how old their parents are. The results are in the stem-and-leaf diagram below.

Ages of the parents	
3	1 2 3 3 6 7 7 8 9
4	0 0 1 1 2 3 4 4 5 5 5 7 8 9 9
5	2 4 9
6	4
tens	units

- Calculate the mean, median and mode of ages of the parents. Round to one decimal place if necessary.
 - For the parents who are 50 years old or older, Alex buys a present for 10 euros. The other parents are getting a present, which is 50 cents cheaper. How much money does Alex spend on the presents?
4. A data set has the following data values: 6, 15, 12, x , 8, 17 and 10.
- The median is 12, what numbers can x be?
 - Can you give a set of data with a median of 12 and mean of 11?

Checklist

- Can you explain the meaning of and the differences between the three different measures of central tendency: the mean, median and mode?
- Can you calculate the mean, median and mode from: 21, 11, 7, 9, 12, 5, 12, 15, 3?
- Can you calculate the mean, median and mode from a certain data set or diagram?
- Can you decide whether the mean, median or mode is most suitable in a given situation?

Lesson 10: Median, Mode and Mean in Frequency tables

Theory

Sometimes we have a lot of data, that it would be too much work to write it all out. However with the help of a frequency table, we can still calculate the mean, mode and median.

955 children were asked how many siblings they have. We see their answers in the frequency table.

Number of siblings	0	1	2	3	4	5
Frequency	203	240	215	121	75	10

X X ✓

240 is the highest frequency, so the mode is 1

Instead of adding 240 1's we can just calculate 240×1 . So we can calculate the **mean**:

$$\text{mean} = \frac{203 \times 0 + 240 \times 1 + 215 \times 2 + 121 \times 3 + 75 \times 4 + 10 \times 5}{955} = 1.448... \approx 1$$

To calculate the median we need to know the middle number of the data. We find this by doing the following steps:

- Find the position of the median:
 If the total amount is *odd*: The median is the middle number. Divide the total amount by 2 and round this value up to find the position.
 If the total amount is *even*: The median is the mean of the two middle numbers. Divide the total amount by 2. The median is the mean of the number on this position and the number on the next position.
- Add frequencies in the table until you get to that position or past it. Draw a red cross underneath the table until you get to the position.

If we divide 955 by 2 we get $477.5 \approx 478$. So the median will be the 478th number. Then we go through the table and add until we get to or past 478. We draw a red mark under 0 and 1, since $203 + 240 = 443$. When we add 2 we get to $203 + 240 + 215 = 658$, which is more than 478. So the **median** is 2.
 In a frequency table it is easy to see what number has the highest frequency. Here 240 is the highest frequency, so the **mode** is 1.

Example

A 2 havo class has the following results for a test. There are 32 students in this classroom. We see the results in a frequency table and a histogram. Calculate the mean, median and mode.

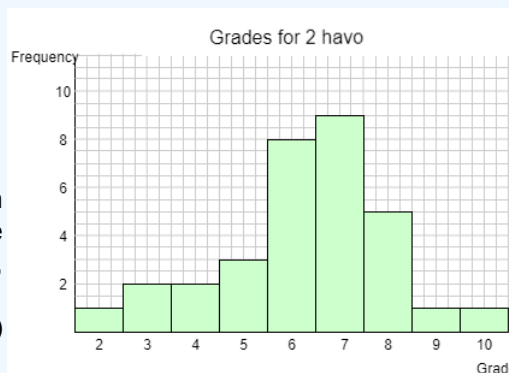
Grade	2	3	4	5	6	7	8	9	10
Frequency	1	2	2	3	8	9	5	1	1

The mean we calculate:

$$\frac{1 \times 2 + 2 \times 3 + 2 \times 4 + 3 \times 5 + 8 \times 6 + 9 \times 7 + 5 \times 8 + 1 \times 9 + 1 \times 10}{32} \approx 6.3$$

We know there are 32 students, so the median will be between the 16th and 17th number. Adding up the frequencies of the grades 2 to 6: $1 + 2 + 2 + 3 + 8 = 16$, so the 16th number is a 6, the 17th number is a 7. Thus the median is 6.5.

The mode we can easily find in the histogram (the highest bar) or in the table (the highest frequency). Here the mode is 7.



Homework

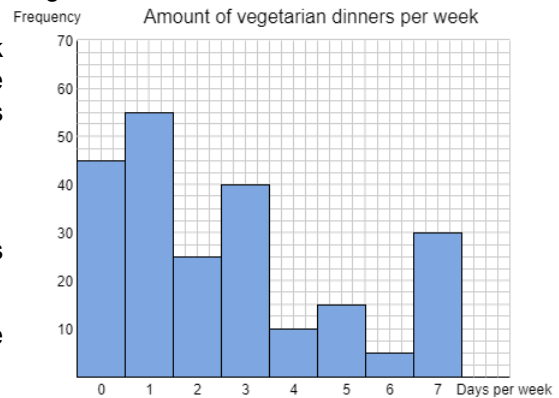
1. Calculate at which position the median is for the given amount of data values.
- (a) 43 (d) 12
 (b) 456 (e) 5
 (c) 397 (f) 23132

2. Tom is doing a study about how many people are in a car on average. During a day he counted how many people were in each car that drove past his house. He has put these values in a frequency table:

Number of people	1	2	3	4	5
Frequency	412	354	212	50	12

- a. How many cars did Tom see?
 b. Draw a histogram using this data. Use steps of 50 on the y-axis.
 c. Calculate the mode, median and mean of the data values. Round to one decimal place if needed.
 d. In which of the three things you just calculated, mode, median or mean, is Tom interested? Explain!
3. Eating vegetarian for a few days each week is happening more and more.

Yara is doing a research into how many days a week people eat vegetarian (so without meat or fish). She has asked 225 students about this and collected this data in a histogram.



- a. What is the mode? How can you tell?
 b. Give the frequency table corresponding to this histogram.
 c. Calculate the median and mean. Round to one decimal place if necessary.
4. Paul is doing a research about the amount of hours 14-year-olds game on a school day. He has made the following frequency table, but he spilled coffee on it. Discuss the following questions with another student.

Hours	0	1	2	3	4	5
Frequency	5	12		23	11	3

- a. If the mode is 3, what do we know about the number underneath the stain? Why?
 b. If the median is 2.5, what do we know about the number underneath the stain? Why?

Checklist

- How can we see what the mode is in a frequency table or in a histogram?
- If there are 403 numbers, can you calculate at which position the median is? Can you find it in a frequency table?
- Do you know a fast way to calculate the mean from a frequency table without just adding all numbers?

★ Exercises

Lesson 1

Calculate the following, round to one decimal place if necessary:

- a. 123% of 230
- b. 0.002% of 75846.3
- c. 1023% of 0.564

Lesson 2

- a. What does it mean when something increases with 150%? What should we write on the place of the question mark below in this case?

$$\text{new amount} = \text{old amount} + ? \times \text{old amount}$$

- b. And what does it mean when something increases with 1000%?

Lesson 3

- a. The temperature during a certain day in November was 8 degrees Celsius. In the night this temperature dropped with 150%. What was the temperature at night?
- b. What does it mean when something decreases with 700%?

Lesson 4

- a. Tim gamed 5 hours in a week. One week later it's May holiday and so he has more time for gaming. During this holiday he games 17 hours in a week. What is the percentage increase in hours of gaming between these two weeks?
- b. The week after the May holiday Tim goes back to gaming 5 hours a week. What is the relative decrease in hours of gaming between this May holiday and the week after that?

Lesson 5

Create a line chart of the following data. Decide for yourself whether or not to use a tear line and how to label the axes.

Table 8: Paris, hottest day

Month	Temperature in °C
April	26
May	28
June	37
July	33
August	31
September	29

Lesson 6

Create a pie chart from the information in the table about a workday of a typical teacher:

Activity	Giving lessons	Drinking coffee	Talking to colleagues	Making tests	Grading homework	Lunch
Time	4:30	0:15	0:45	1:15	0:45	0:30

Note that 4:30 corresponds to 4 hours and 30 minutes.

Lesson 9

We have the data set: 4, 6, 2, 4, x, 9.

We know that the mean is 5.5. What should x be?

Lesson 10

3242 students have been asked how many siblings they have. Their answers can be found in the frequency table below:

Number of siblings	0	1	2	3	4
Frequency	579	1042	812	484	325

Calculate the mode, median and mean. Round the mean to one decimal place.

Diagnostic Test

1. We call mountains higher than 14000 feet (4267 m) 'fourteeners'. There are 96 fourteeners in the United States, of which 53 in Colorado. What percentage of fourteeners in the United States are in Colorado? Round your answers to one decimal place.
2. The average mathematics grade of 2 havo students was a 6.4 in 2018. In 2019 it decreased by 9%. Round grades and percentages to one decimal place.
 - a. What is the average mathematics grade in 2019?
In 2020 it increased again with 7%.
 - b. What is the average mathematics grade in 2020?
In 2021 the average grade is a 6.6.
 - c. With what percentage did the grade change in 2021 compared to 2018?
 - d. With what percentage did the grade change in 2021 compared to 2020? Did it increase or decrease?
3. Yara has calculated the following:

$$\text{new amount} = 45 - 0.3 \times 45.$$

Think of a real life example where this needs to be calculated.

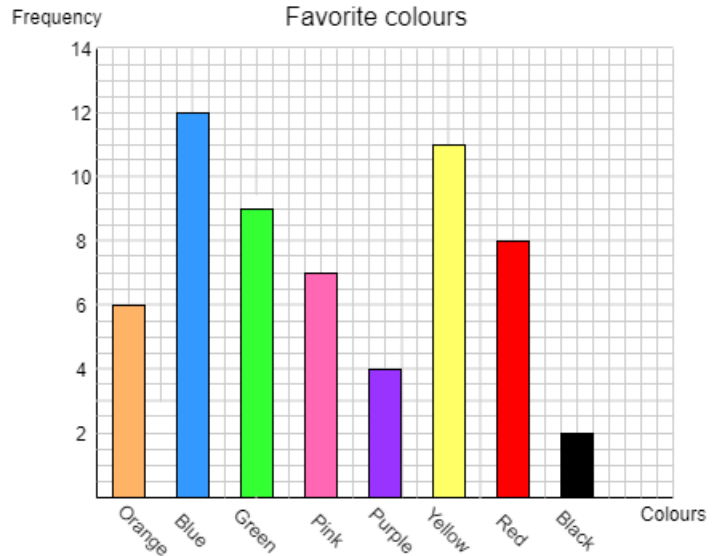
4. The amount of toilet paper packages a supermarket has sold in the first six months of 2020 are given in the table below:

Month	Number of toilet paper packages sold $\times 100$
January	41
February	40
March	97
April	66
May	59
June	61

- a. Use a line chart to visualise the data in the table. Let 1 cm represent 500 packages and use a tear line.
- b. What is the percentage increase in number of toilet paper packages sold from February 2020 to March 2020?
- c. What is the mean number of toilet paper packages the super market sold in a month in these first six months of 2020? Round to a whole number.

5. Given the bar chart with the favourite colours of a group of children.

- Create a frequency table corresponding to this bar chart.
- What percentage of the children have their favourite colour as blue or orange? Round to one decimal place.
- What percentage of children have their favourite colour as green, purple or black? Round to one decimal place.
- Rose says 'More children like blue, yellow or green than all the other colours combined'. Is she correct? Why or why not?



6. 29 students named the cities they live in, Their answers are given in the frequency table.

City	Eindhoven	Son	Nuenen	Best	Other
Frequency	13	7	3	5	1

- How would you explain to another student how to draw a pie chart?
 - Draw a pie chart for this data using your steps explained in a.
7. In the following double stem-and-leaf diagram we see the travelling time to school (one way) for two classes, 2A and 2B.

Travelling time class 2A			Travelling time class 2B	
9 7 4 4	0		5 5 8	
5 5 3	1		1 4 4 4 4	
2 1 1 1	2		3 3 6	
6	3		2	
1	4			
units	tens		units	

Table 9: Double stem-and-leaf diagram for travelling times

- Give the mode for both groups.
- Give the median for both groups.
- Give the mean for both groups. Round to two decimal places.
- Which central tendency do you think would be best to compare these two groups? Explain!

Overview of Terms

English term	Dutch term
Absolute decrease	Afname in hoeveelheid
Absolute increase	Toename in hoeveelheid
Amount	Hoeveelheid
Bar chart	Staafdiagram
Central angle	Hoek gemeten bij het middelpunt van een cirkeldiagram
Chart	Grafiek
Class boundaries	Klassengrenzen
Class intervals	Klassen
Data	Data (verzamelde informatie over een onderwerp)
Data set/Set of data	Verzameling van data
Data values	Waarden
Decrease	Afname
Discount	Korting
Formula	Formule
Frequency	Frequentie
Frequency distribution	Klassenindeling
Frequency table	Frequentietabel
Histogram	Histogram
Increase	Toename
Interval width	Klassenbreedte
Line chart	Lijndiagram
Mean (Average)	Gemiddelde
Measures of central tendency	Centrummaten
Median	Mediaan
Mode	Modus
Percentage	Percentage
Pie chart	Cirkeldiagram
Position (the ..th number in the row)	Positie (plek in een rij)
Relative decrease	Afname in procenten
Relative increase	Toename in procenten
Sectors	Sectoren
(Double) Stem-and-leaf diagram	(Dubbel) Steel-bladdiagram
To tally	Turven
Tear line	Scheurlijn
Ten (stem in a stem-and-leaf diagram)	Tientallen
Unit (leaf in a stem-and-leaf diagram)	Eenheden
Visualisation	Visualisatie (iets laten zien met een beeld)

Teacher's Manual

Introduction

This Teacher's Manual provides background information for the lesson material Chapter 6; Percentages and Diagrams. This chapter was constructed by Lindsey van der Aalst, Jolijn Cottaar, Brenda Krekels, Erwin Nevels and Iris Portegijs, master students Science Education and Communication from the University of Technology Eindhoven (TU/e) for the course EME32, Betadidactisch ontwerpen. This project was commissioned by Stedelijk College Eindhoven and supervised by dr. Alexander Schüler-Meyer from the TU/e.

In this teacher's manual, all necessary information needed for the teacher to prepare and give the lesson is given per lesson. The learning goals for the lessons are given, as well as the important terms that the students should know at the end of the lesson. If there is preparation needed for the lesson, this is explained, and a suggestion for an activation idea for the lesson is given. You are of course free to come up with your own activation idea, or to alter this idea so it fits your class best.

The answer manual for all the homework exercises and Diagnostic Test is included as well as Star exercises and their answers. These Star exercises can be used for differentiation in your lessons, as they are well suited for students who need to be challenged.

Lesson 1

Learning Goals

1. Students know what percentages mean (part of a whole, whole = 100%)
2. Students can calculate how much a given percentage of a given whole is (with the whole is not 100, i.e. the whole is 500 euros)
3. Students can calculate the percentage of a given part and a given whole (with the whole is not 100)
4. Students know how to round percentages

Preparation

No preparation required.

Activation Idea

Discuss with students where one would find percentages and what they are used for. You can for example do this by showing them a recent (English) newspaper article.

Important Terms

Check if the students know and understand the following terms both in Dutch and English:

- Percentage

Lesson 2

Learning Goals

1. Students can reproduce the formula:
new amount = old amount + (percentage increase as decimal number) x old amount
2. Students can calculate the new whole for a given percentage increase with the formula mentioned above
3. Students can calculate the new whole for a given percentage increase when these are given in text
4. Students can understand texts about percentage increase
5. Students can discuss percentage increase

Preparation

No preparation required.

Activation Idea

Find an example of a product that is now available in larger size (pizzas, potato chips, Unox can, etc.). Let student discuss other types of products that have this increase in size/content and write these down or show the pictures of these products on the screen. Ask the students what the difference between the current and the old product is (the students can conclude that the current size is more than the old size, i.e. the size has increased).

Important Terms

Check if the students know and understand the following terms both in Dutch and English:

- Amount
- Formula
- Increase

Lesson 3

Learning Goals

1. Students can reproduce the formula:
new amount = old amount - (percentage decrease as decimal number) x old amount
2. Students can calculate the new whole for a given percentage decrease with the formula mentioned above
3. Students can calculate the new whole for a given percentage decrease when these are given in text
4. Students can understand texts about percentage decrease
5. Students can discuss percentage decrease

Preparation

No preparation required.

Activation Idea

Show a photo of sales (so in a shopping street, in advertisements, sales in online clothing sites, sales for games, etc.). Let the students discuss these and add more discount. What happens with the price of the product? (this decreases, i.e. the new price is lower than the old one).

Important Terms

Check if the students know and understand the following terms both in Dutch and English:

- Decrease
- Discount

Lesson 4

Learning Goals

1. Students can calculate the percentage increase for a given old and new amount with the formula:
percentage increase = $\frac{\text{increase}}{\text{old amount}} \times 100\%$
2. Students can calculate the percentage decrease for a given old and new amount with the formula:
percentage decrease = $\frac{\text{decrease}}{\text{old amount}} \times 100\%$
3. Students can calculate the percentage increase or decrease for a problem explained in a text
4. Students can understand texts about percentage increase and decrease
5. Students can discuss percentage increase and decrease

Preparation

No preparation required.

Activation Idea

Establish the connection with the theory from the previous lesson. For example, first ask the students how much money they would have on their savings account next year if they would set 90 euros aside and the bank offers 1.5% interest. The students should be able to come up with the following calculation:

$$90 + 0.015 \times 90 = 91.35 \text{ euros}$$

This activates the prerequisites of the students and it is a check if the students understood/remember the previous lesson.

Thereafter, tell students that today you will show them how to calculate the amount of interest the bank offers when you know the old and new amount of money on your savings account.

This can for example be shown using the following example: Suppose that (again) you have set aside 90 euros on your savings account. You need 92 euros to buy a used PlayStation. How much interest does a bank need to offer to be able to buy this PlayStation in a year?

Important Terms

Check if the students know and understand the following terms both in Dutch and English:

- Absolute decrease
- Absolute increase
- Relative decrease
- Relative increase

Lesson 5

Learning Goals

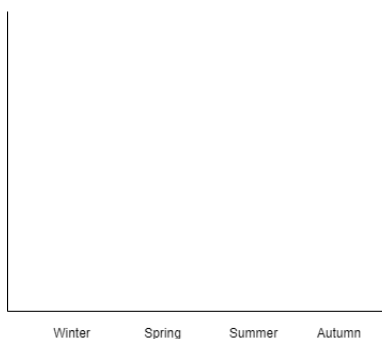
1. Students can explain why it is useful to visualise data
2. Students can recognise a bar chart when they see one
3. Students can explain the characteristics of a bar chart (length of bars, label of bars, separate bars)
4. Students can interpret information (for example amounts) from a bar chart
5. Students can draw a bar chart
6. Students can recognise a line chart when they see one
7. Students can explain the characteristics of a line chart (data points connected with straight lines)
8. Students can interpret information from a bar chart
9. Students know what a tear line is and how and when you use it
10. Students can draw a line chart (with information and values on the axis)
11. Students can explain the information shown in a line chart of bar chart
12. Students can explain the differences between bar charts and line charts

Preparation

If possible, already draw the template for the bar chart on the board or on the online whiteboard. Furthermore, make sure you have post-its (or something similar) available.

Activation Idea

Hand out post-its and draw the following template for a bar chart on the board:



Let students write their month of birth on their post-it and also the season that corresponds to this month. Let them stick their post-its (above each other) above the correct season of their birth. Now they created their own bar chart.

NB: In case of hybrid lessons you can choose to only use the birth months of the people that are in the classroom or you can let the students that follow the class online type their birth month in the chat and write this on a sticky note yourself. You can also use an online whiteboard (e.g. Microsoft Whiteboard) to create the bar chart to increase the visibility for the students at home. You can even give students permission to this whiteboard and let them add a coloured text box with their birth month in it above the corresponding season. In this way, both the students at home and the students in the class can create the bar chart.

Another option is to partner up the students at home with the ones in the classroom and have the student in the classroom fill in two post-its.

Important Terms

Check if the students know and understand the following terms both in Dutch and English:

- Bar chart
- Chart
- Data values
- Line chart
- To tally
- Tear line
- Visualisation

Lesson 6

Learning Goals

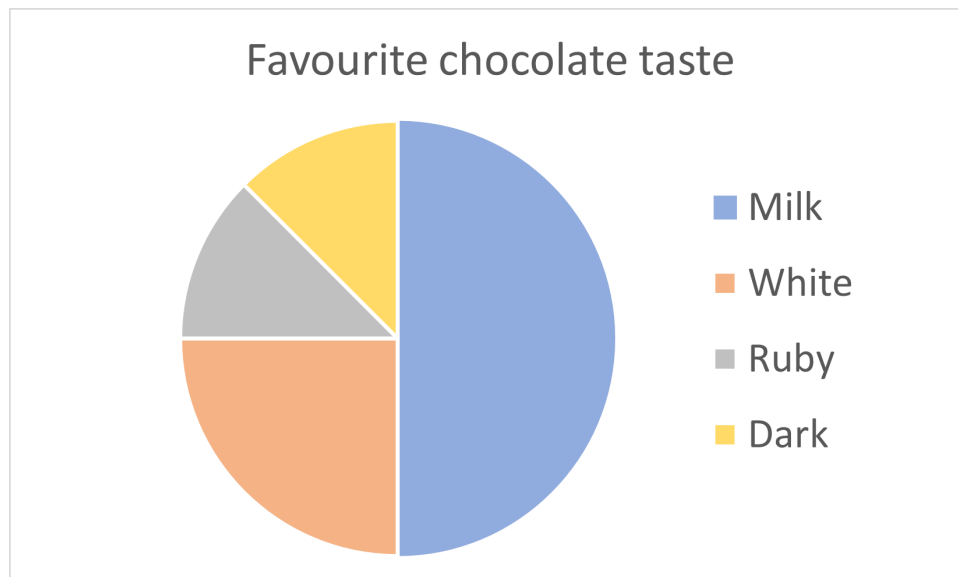
1. Students know how to read and interpret a pie chart.
2. Students know that the sum of the angles at the centre of a pie chart is 360 degrees.
3. Students know what sectors are in a pie chart and how to calculate the central angle of a sector in a pie chart.
4. Students can calculate the central angle of a sector given the corresponding percentage.
5. Students can represent data in a pie chart by following the four steps of 'How to draw a pie chart'.
6. Students can explain the data represented in a pie chart.
7. Students can explain the characteristics of a pie chart.

Preparation

No preparation required.

Activation Idea

Show the students the following pie chart. Let the students discuss with each other what this figures actually shows. Then, indicate that 40 people have provided input and let the students make a corresponding bar chart. In this way, the students are already introduced to the pie chart and see the connection with the previous lesson.



Important Terms

Check if the students know and understand the following terms both in Dutch and English:

- Central angle
- Legend
- Pie chart
- Sectors

Lesson 7 & 8

Project

These two lessons can be given as a project. In this project, the students will learn how to visualise data in frequency tables, histograms and stem-and-leaf diagrams. At the end of these two lessons, they should also be able to explain and read these diagrams.

Learning Goals

1. Students know what frequency and frequency distribution is
2. Students know the meaning of classes, class intervals, class boundaries and interval width
3. Students know how to read, interpret and explain data in; a frequency table, a histogram and a stem-and-leaf diagram
4. For a given data set, students can make; a frequency table, a histogram and a stem-and-leaf diagram
5. Students can explain the differences and relations between the data, the frequency table, histogram, stem-and-leaf diagram and frequency distribution

Preparation

Before the start of the first project lesson, ask every student for two things (for example using the Google Forms provided):

1. How many hours do the students spent on hobbies per week? Make sure they round it down to hours.
2. How long is their commuting time in minutes? So how many minutes does it take the student to travel from home to school? Do they travel by bike or by some other form of transportation?

Gather this information for every student and make data sets for the commuting time (split the data in commute by bike and other forms of transportation, student will make a double stem-and-leaf diagram with this data) and for the hobby-related activities. Split the data sets in data sets A and B for both topics, these smaller data sets should have about 15 data points (if you have a small class, reuse some data points). If you can, try to split the data sets in such a way that the smaller data sets still have varying values but will have slightly different averages (so the difference in the frequency tables and histograms will be evident).

Then split your students in heterogeneous groups of about 3 students (so mixed levels of both mathematics and language skills). Prepare the data sets A for half the groups and B for the other groups by putting all the data points of a data set in one file. Do not provide relations or order in these data sets yet, but keep it random. So for example for data set A for the hobbies: 5 4 3 4 2 8 6 2 7 8 4 3 1 6 1

If possible try to shift tables in the classroom in such a way that the groups can sit together. Make sure that there is plenty of time for the discussions. You can do this for all topics at the end of the second lesson or do an discussion at the end of the first lesson as well. If the lesson is hybrid, use breakout rooms or let the students call each other/use some other form of communication (but preferably something that requires speech). In the hybrid form, students can swap their exercises by sending a picture.

Activation Idea

Discuss the data that you gathered, without giving away too much information. Try to make general remarks; "I could see that many of you have a lot of hobbies!" or "About half of you have to travel longer than I do."

OR:

Discuss what the use of data is (which instances gather data about you, what do they do with that information) and what the word data can mean (data (ENG & NL); MBs used to access the internet, data (ENG & NL); plural of date(ENG) or datum(NL), data (ENG & NL): information about a certain topic that was gathered, so this can be age, name, grade,...)

OR:

Talk about burn-outs, time spent on recreation and the importance of relaxation/ the danger of constant stress. Explain that they will be researching this data, to see what specific things they can conclude about the data sets.

Important Terms

Check if the students know and understand the following terms both in Dutch and English:

1. Class boundaries
2. Class intervals
3. Data (meaning information about a certain topic that was gathered)
4. Data set/Set of data
5. Frequency
6. Frequency distribution
7. Frequency table
8. Histogram
9. Interval width
10. (Double) Stem-and-leaf diagram
11. Ten (stem in a stem-and-leaf diagram)
12. Unit (leaf in a stem-and-leaf diagram)

Furthermore, the terms *commute* and *recreation* are used in this project. These terms are mathematically not relevant, but it might be wise to check if the students indeed understand these terms.

Lesson 9

Learning Goals

1. Students know the meaning of and the differences between *mean*, *median* and *mode* and they know that those three form the *measures of central tendency*
2. Students know how to calculate the mean, median and mode from a certain data set or diagram
3. Students can decide whether the mean, median or mode is most suitable in a given situation
4. Students can discuss differences and relevance of mean, median and mode

Preparation

No preparation required.

Activation Idea

Show the data for the three ideas from the theory (pocket money). Discuss which idea (Sophie's, Aisha's or Peter's) is best suited to compare the pocket money of the two groups?

Three students, Sophie, Aisha and Peter want to compare the pocket money of a group of 9 boys and a group of 8 girls. They each have their own idea about how to compare this. They ask each of the boys and the girls to tell them the amount of pocket money they get every week.

<p><u>Sophie's idea</u> We order the amounts of pocket money from low to high for the boys and the girls. Then we compare the middle number of the boys with the middle number of the girls</p>	<p><u>Aisha's idea</u> We compare the amounts of pocket money that is given most often mentioned by the girls with the one that most often mentioned by the boys.</p>	<p><u>Peter's idea</u> We take all the pocket money of the boys, add it up and share it among the boys. We do the same for the girls and we compare these shares.</p>
---	---	---

Important Terms

Check if the students know and understand the following terms both in Dutch and English:

- Mean (Average)
- Measures of central tendency
- Median
- Mode

Lesson 10

Learning Goals

1. Students know the meaning of and the differences between total frequency, absolute frequency and relative frequency
2. Students can calculate the total frequency based on a frequency table
3. Students can read the absolute frequency for a certain data value from a frequency table
4. Students can calculate the relative frequency of a certain data value based on a frequency table
5. Students can reproduce the formula for calculating the relative frequency (i.e. $\text{relative frequency} = \frac{\text{absolute frequency}}{\text{total frequency}} \times 100\%$) and can use this formula in calculations
6. Students can calculate/read the mean, median and mode from a frequency table
7. Students can discuss differences and relevance of mean, median and mode in a frequency table

Preparation

It is important that the students understand frequency tables. It might be useful to prepare a recap of this topic if you feel like the students don't understand this yet.

Activation Idea

Recreate the data points of the theory (amount of siblings) with the students. Create a frequency table together, calculate the mean, median and mode with them the old way. Then introduce the faster way.

Important Terms

Check if the students know and understand the following terms both in Dutch and English:

- Position (the ..th number in the row)

Lesson 11

Learning Goals

1. Prepare for final test for example using the diagnostic test.

Preparation

-

Activation Idea

Construct an overview of all the learned ideas with the students.

Important Terms

-

Lesson 12

Learning Goals

1. Prepare for final test

Preparation

-

Activation Idea

Test the knowledge of the students of terms and charts with for example a Kahoot! quiz.

Important Terms

-

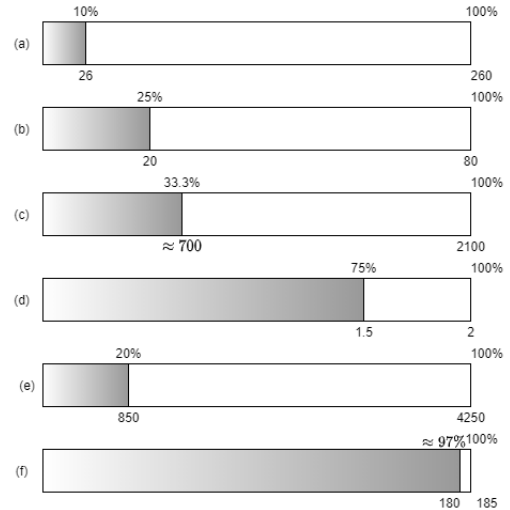
Answer Manual

Lesson 1

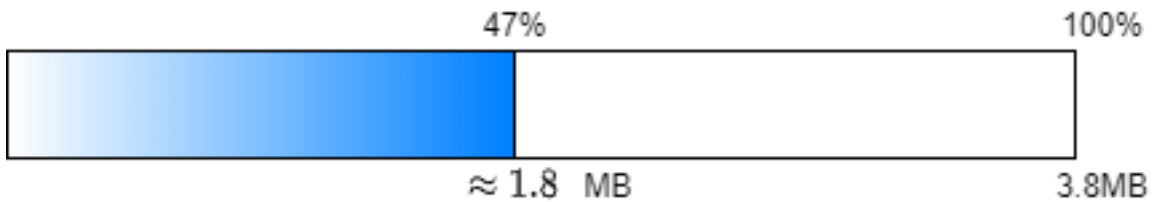
Homework

1. Calculations below, bars to the right. Both need to be given.

- a. $0.10 \times 260 = 26$
- b. $0.25 \times 80 = 20$
- c. $0.333 \times 2100 = 699.3$
- d. $\frac{1.5}{2} \times 100\% = 75\%$
- e. $\frac{850}{4250} \times 100\% = 20\%$
- f. $\frac{180}{185} \times 100\% \approx 97.3\%$



- 2. a. She has already downloaded $0.7 \times 18 = 12.6$ GB. So she still needs to download $18 - 12.6 = 5.4$ GB.
- b. $\frac{3.6}{18} \times 100\% = 20\%$.
- 3. a. I-B, II-A, III-C
- b. I: $0.28 \times 5 = 1.4$ GB
 II: $0.85 \times 5 = 4.25$ GB
 III: $0.46 \times 5 = 2.3$ GB
- 4. $0.47 \times 3.8 = 1.786$ MB has already been downloaded.



5. Achmed is right. Lucy is calculating how much 35 out of 200 is instead of how much 35% of 200 GB is.

Checklist

-
- $\frac{1}{2} \times 100\% = 50\%$, thus at 50%.
- $0.25 \times 4 = 1$ GB has been downloaded.

★ Exercise

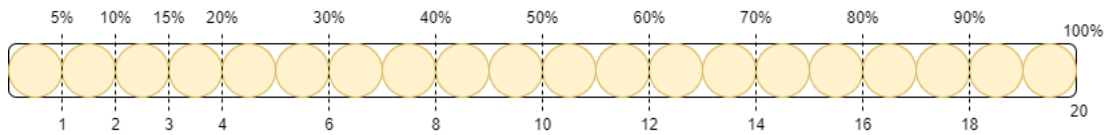
- a. $\frac{123}{100} \times 230 = 282.9$
- b. $\frac{0.002}{100} \times 75846.3 \approx 1.5$

c. $\frac{1023}{100} \times 0.564 \approx 5.8$

Lesson 2

Homework

1. There are multiple ways to split up the bar. This is one of the possibilities. First split up the bar into ten equal parts, each 10%. Then split the first and second part each into two parts of each 5%:



So we need to add three candies.

Calculation: new amount = $20 + 0.15 \times 20 = 20 + 3 = 23$

2. There are multiple ways to split up the bar, this is just one option.

a. new amount = $20 + 0.05 \times 20 = 20 + 1 = 21$

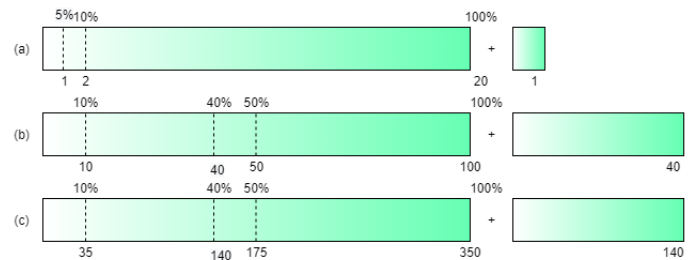
b. new amount = $100 + 0.4 \times 100 = 100 + 40 = 140$

c. new amount = $350 + 0.4 \times 350 = 350 + 140 = 490$

d. new amount = $59 + 0.75 \times 59 = 59 + 44.25 \approx 103.3$

e. new amount = $823 + 0.025 \times 823 = 823 + 20.575 \approx 843.6$

f. new amount = $32.5 + 0.09 \times 32.5 = 32.5 + 2.925 \approx 35.4$



3. a. new amount = $1500 + 0.03 \times 1500 = 1500 + 45 = 1545$, so there are 1545 students in 2021 who follow the Dutch program.

b. we have to calculate 11 out of 416. $\frac{11}{416} \times 100\% \approx 2.6\%$. So about 2.6% is new.

4. new amount = $0.55 + 0.2 \times 0.55 = 0.55 + 0.11 = 0.66$ pound.

5. TiresBrakes costs: new amount = $650 + 0.20 \times 650 = 650 + 130 = 780$ euros.

BikesCo costs: new amount = $720 + .10 \times 720 = 720 + 72 = 792$ euros.

So Ali is right, since the TiresBrakes bike costs 12 euros less than the BikesCo bike.

Checklist

- The new price is: $20 + 0.1 \times 20 = 20 + 2 = 22$ euros.

★ Exercise

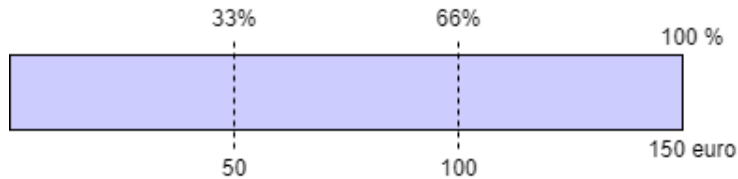
- a. This means that one and a half times the amount you had is added to the old amount. Instead of the question mark we should write $\frac{150}{100} = 1.5$.

- b. This means that 10 times the amount you had is added to the old amount. So the amount you had gets 11 times as large.

Lesson 3

Homework

1. We can split up 100% into 3 equal part. Each part will be a little more then 33% but it is very close. So it will be approximately 100 euros.



Calculation: new amount = $150 - 0.33 \times 150 = 150 - 49.5 = 100.5$ euros.

2. a. new amount = $40 + 0.08 \times 40 = 40 + 3.2 = 43.2$
b. new amount = $100 - 0.17 \times 100 = 100 - 17 = 83$
c. new amount = $600 - 0.4 \times 600 = 600 - 240 = 360$
d. new amount = $59 + 0.24 \times 59 = 59 + 14.16 \approx 73.16$
e. new amount = $645 - 0.025 \times 645 = 645 - 16.125 \approx 628.9$
f. new amount = $86.3 + 0.16 \times 86.3 = 86.3 + 13.808 \approx 100.1$
3. a. new amount = $200 - 0.4 \times 200 = 200 - 80 = 120$ grams. So a bag of crisps now weighs 120 grams.
b. new amount = $7.50 - 0.05 \times 7.50 = 7.50 - 0.375 \approx 7.13$ euros. So that month costs 7.13.
4. a. new amount = $4.5 - 0.23 \times 4.5 = 4.5 - 1.035 = 3.465$. So he games for 3 hours and $0.465 \times 60 \approx 28$ minutes.
b. 3 hours out of 24 hours : $\frac{3}{24} \times 100\% = 12.5\%$. So she spent 12.5% of her Sunday on her mathematics homework.
c. new amount = $1 + 1.2 \times 1 = 1 + 1.2 = 2.2$. So Damian spends 2 hours and $0.2 \times 60 = 12$ minutes watching television on Saturday.
5. Misha claims that for both games they have to pay $80 - 0.5 \times 80 = 40$ euros, which is half of what they would originally pay for two games. The discount is only 25% so that cannot be right. They should pay more than that. Stan claims that the price of one game can be calculated with new amount = $40 - 0.25 \times 40 = 30$ euros. So for both games, Stan and Misha will pay $2 \times 30 = 60$ euros. Stan is right.

Checklist

- 33% of 3 euro is 1 euro. So your favourite chocolate is now $3+1 = 4$ euro.
Calculation: New amount = $3 + 0.33 \times 3 = 3 + 1 = 4$.
- 20% of 25 euros is 5 euro. So the sweater now costs $25 - 5 = 20$ euro.
Calculation: New amount = $25 - 0.2 \times 25 = 25 - 5 = 20$.

★ Exercise

- a. $8 - 1.5 \times 8 = 8 - 12 = -4$ degrees Celsius.
- b. This means that 7 times the amount you had is subtracted from the old amount. The new amount is 6 times the old amount with a minus sign in front of it.

Lesson 4

Homework

- 1-B 2-D 3-A 4-C
- Relative decrease is $\frac{164-162}{162} \times 100\% \approx 12.3\%$
 - Percentage increase is $\frac{170-162}{162} \times 100\% \approx 4.9\%$. So Justin needs to grow 4.9%.
 - 4.9% is more than 4%, so it has become unlikely.
 - *
 - *
- In the first situation the download is at $\frac{1200}{2000} \times 100\% = 60\%$.
In the second situation the download is at $\frac{1600}{2000} \times 100\% = 80\%$.
 - Old amount = 1200 and the increase is $1600 - 1200 = 400$ MB. So the percentage increase is $\frac{400}{1200} \times 100\% \approx 33.3\%$.
 - The difference between the download bars is $80-60=20\%$, while the percentage increase calculated in part a is 33.3%. These are not the same, because the difference in percent between the download bars is the absolute increase in percent between the two download bars and not the relative increase in MB.
- old amount = 1 and the increase is 0.20 euro. So the relative increase is $\frac{0.20}{1} \times 100\% = 20\%$.
 - The roses will cost less than 1 euro, since we have to subtract 20 % of 1.20 euro, which is more than 0.20 euro.
 - The new price is $1.20 - 0.20 \times 1.20 = 0.96$ euro per rose. This is indeed less than 1 euro.

Checklist

- The absolute increase = new amount – old amount = $9-7 = 2$ euros.
- The relative increase = $\frac{\text{increase}}{\text{old amount}} \times 100\% = \frac{2}{7} \times 100\% \approx 28.6\%$.
- The absolute decrease = new amount – old amount = $14-11 = 3$ euros.
- The relative decrease = $\frac{\text{decrease}}{\text{old amount}} \times 100\% = \frac{3}{14} \times 100\% \approx 21.4\%$.

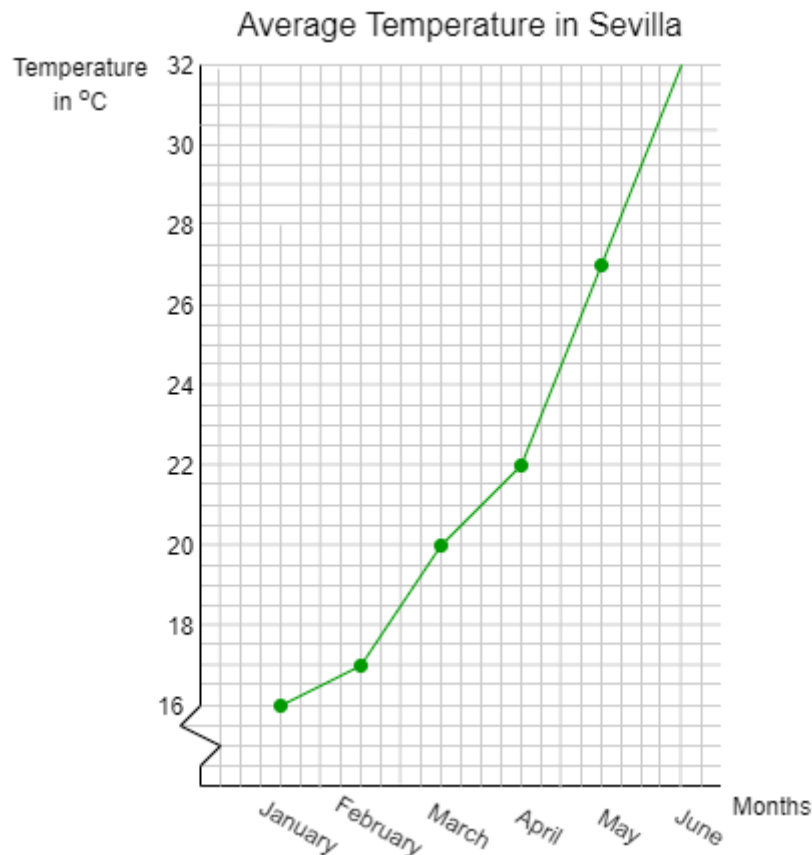
★ Exercise

- The increase is $17 - 5 = 12$ hours and the old amount is 5. So percentage increase = $\frac{12}{5} \times 100\% = 240\%$
- The decrease is $17 - 5 = 12$ hours and the old amount is 17. So percentage decrease = $\frac{12}{17} \times 100\% = 70.1\%$.

Lesson 5

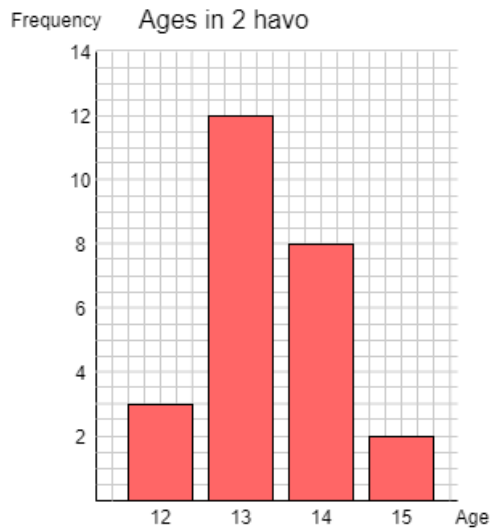
Homework

- Tower Bridge: 11 million.
Eye of London: 9 million.
Museum of Natural History: 4.5 million.
Big Ben: 12 million.
Buckingham Palace: 10 million.
 - In 2018 there were 12 million visitors of Buckingham Palace, since there were 12 million visitors of the Big Ben in 2019. In 2019 there were 10 million visitors of Buckingham Palace. So the decrease is $12 - 10 = 2$ million visitors. The old amount is 12 million, so the percentage decrease is $\frac{2}{12} \approx 16.7\%$.
 - 4.5 million tourists visited the museum, 2 million were students. So this means 2.5 million tourists were not students.
 $\frac{2.5}{4.5} \times 100\% = 55.6\%$. So 50% were not students from the visitors of the Museum of Natural History.
- The line chart should like:



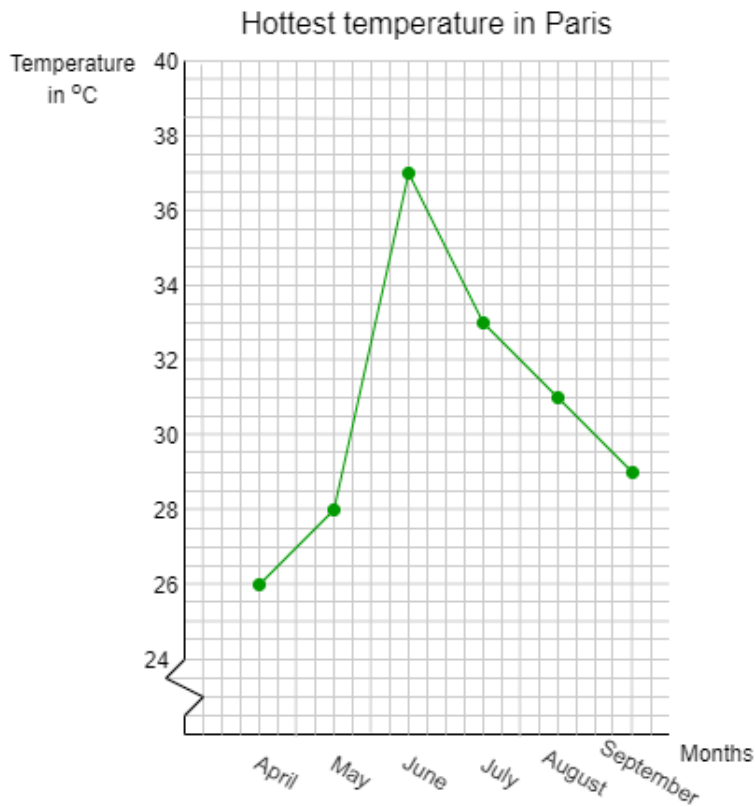
- The increase is $22 - 16 = 6$ degrees Celsius and the old amount is 16. So the percentage increase is $\frac{6}{16} \times 100\% = 37.5\%$.
 - Yes, you could also use a bar chart for this since you have all the data you need for this.
- Jasper: in 2013 about 650 million tablets were sold. In 2018 about 1200 million tablets were sold.
 $1200000000 : 650000000 \approx 1.8$ so Jasper is not correct.
Li-Ming: We cannot know this, since only the total sale of tablets per year is given.
Nina: Yes, in 2020 worldwide about 1 300 million tablets were sold, while in 2013 it was about 650 million.
So it has increased.

4. The bar chart should look like:



★ Exercise

The line chart should look like :



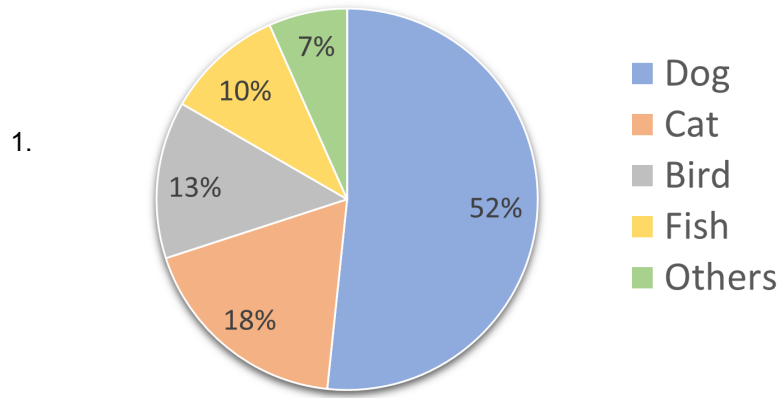
Using a tear line is a good idea, since we start only at 26 degrees. And making steps of two makes sure we can still easily read the chart.

Lesson 6

Homework

Favourite pet

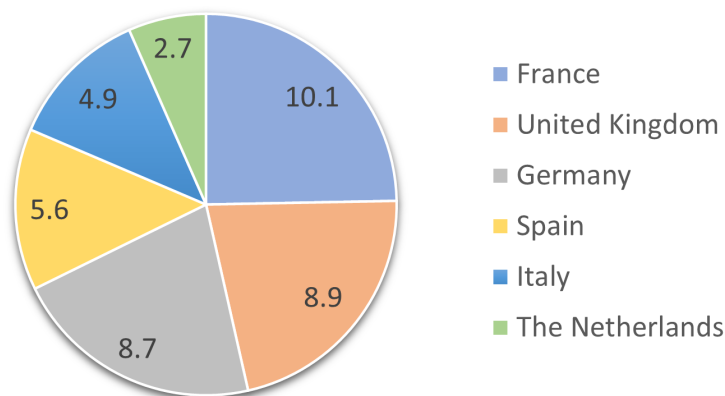
total: 60 people



2. a. $\frac{14}{10} \times 72 = 10$ people went on holiday by train.
 b. $\frac{12}{74} \times 100\% \approx 16.2\%$. So 16.2% went on holiday by bus.
 c. $\frac{32}{100} \times 360^\circ \approx 115^\circ$
3. a. $40.9 - 10.1 - 8.9 - 5.6 - 4.9 - 2.7 = 8.7$. So there were 8.7 million downloads in Germany.
 b. Germany: $\frac{8.9}{40.9} \times 360^\circ \approx 78^\circ$.
 Spain: $\frac{5.6}{40.9} \times 360^\circ \approx 49^\circ$.
 Italy: $\frac{4.9}{40.9} \times 360^\circ \approx 43^\circ$.
 The Netherlands $\frac{2.7}{40.9} \times 360^\circ \approx 24^\circ$.

Number of downloads on Netflix

total: 40.9 million



- c. $\frac{10.1}{40.9} \times 100\% \approx 24.7\%$. So 24.7% of the downloads were in France.
4. We can calculate:
 Chinese food in group A: $\frac{20}{100} \times 30 = 6$
 Fries in group B: $\frac{24}{100} \times 30 = 6$
 So neither is right, since there exactly the same number of people in group A who like Chinese food as in group B that like fries.

5. We calculate:

Sleeping: $\frac{9}{24} \times 360^\circ \approx 135^\circ$

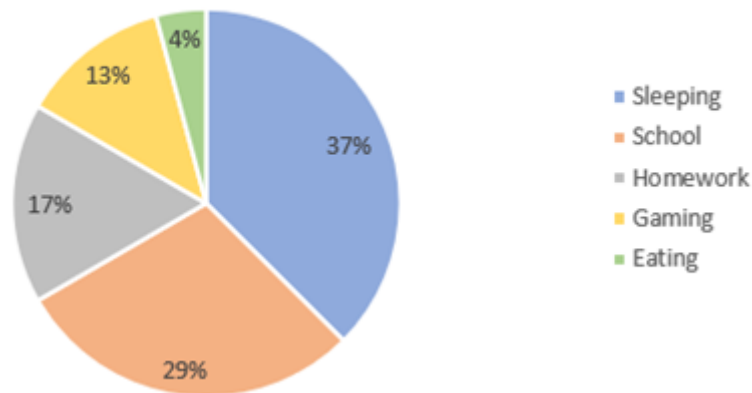
School: $\frac{7}{24} \times 360^\circ \approx 105^\circ$

Homework: $\frac{4}{24} \times 360^\circ \approx 60^\circ$

Gaming: $\frac{3}{24} \times 360^\circ \approx 45^\circ$

Eating: $\frac{1}{24} \times 360^\circ \approx 15^\circ$

Simon's schoolday



Checklist

-
- The central angle is $\frac{6}{12} \times 360^\circ = 180^\circ$.
- The central angle is $\frac{25\%}{100\%} \times 360^\circ = 90^\circ$.
- The amount of people is $\frac{72}{360} \times 50 = 10$ people.

★ Exercise

The workday consists of 8 hours in total (you find this by adding up all the times for the different activities).

The central angles we need to use are:

Giving lessons: $\frac{4.5}{8} \times 360^\circ \approx 203^\circ$ (Note that 4 hours and 30 minutes corresponds to 4.5 hours.)

Drinking coffee: $\frac{0.25}{8} \times 360^\circ \approx 11^\circ$ (Note that 15 minutes corresponds to 0.25 hours.)

Talking to colleagues: $\frac{0.75}{8} \times 360^\circ \approx 34^\circ$ (Note that 45 minutes corresponds to 0.75 hours.)

Making tests: $\frac{1.25}{8} \times 360^\circ \approx 56^\circ$ (Note that 1 hour and 15 minutes corresponds to 1.25 hours.)

Grading homework: $\frac{0.75}{8} \times 360^\circ \approx 34^\circ$ (Note that 45 minutes corresponds to 0.75 hours.)

The remaining part of the pie chart should then be the sector for lunch.

Lesson 9

Homework

- Data values in order: 2.50 3 3 5.50 (6) 6 6 9 10
The median is the middle number. There are 9 numbers, so the middle number is the 5th number, which is 6.
The mode is 6, since this is the most frequent number.
The mean is $\frac{10+3+6+2.50+3+6+9+6+5.50}{9} \approx 5.6$
So for Sophie and Aisha's idea the boys have more pocket money, Peter's idea says the girls have more pocket money.
- $\frac{15+18+16+14+15+15+12+17+90+95}{10} = 30.7$ So the mean salary is 30700 euros.
 - Most workers have salaries in the €12000 to €18000 range. The mean is very much affected by the two largest salaries. So the mean is way higher than the salaries that the most workers have. Therefore, in this situation, we would like to have a better measure of central tendency.
 - The median would be a better measure of central tendency in this situation.
 - If we order the salaries in thousand euros, we get 12 14 15 15 15 16 17 18 90 95. There are 10 numbers, so the median is in the middle of the fifth and sixth number, so in the middle of €15000 and €16000. So the median is €15500. The mode = €15000, since this is the most frequent salary.
- Mean is $\frac{31+32+33+33+36+37+37+38+39+40+40+41+41+42+43+44+44+45+45+45+47+48+49+49+52+54+59+64}{28} \approx 43.1$.
The median is the mean of the 2 middle values (the 14th and 15th value): 42 and 43. So the median is $\frac{42+43}{2} = 42.5$.
The mode is 45, since this is the number that appears most frequently..
 - There are 4 parents that are 50 years old or older. So Alex spends $4 \times 10 + 24 \times 9.50 = 40 + 228 = 268$ euros.
- Data values in order: 6 8 10 12 15 17 (and x)
If the median is 12, then 12 is the middle (=4th) data value. So x can be 12 or higher.
 - There are many options. For example: 8, 12, 13.

Checklist

-
- The mean is $\frac{21+11+7+9+12+5+12+15+3}{9} \approx 10.6$.
The median is 11, since it is the middle number.
The mode is 12, since it occurs twice.
-
-

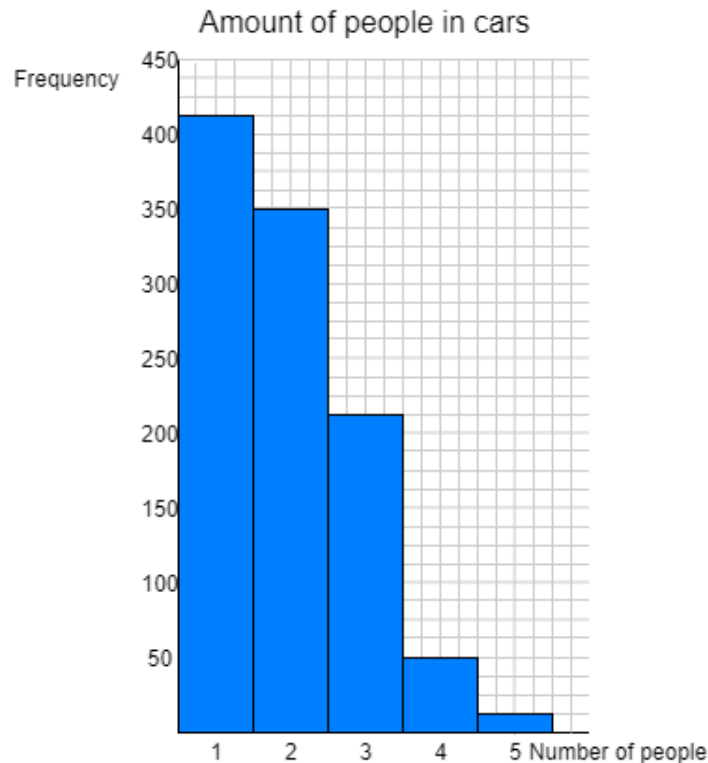
★ Exercise

The mean is $\frac{4+6+2+4+x+9}{6} = \frac{25+x}{6}$. This should be equal to 5.5. You could try out different values for x to check when this is indeed the case. Then you should find that $x = 8$.

Lesson 10

Homework

1. a. $\frac{43}{2} = 21.5$ so the median is the average of the 21st and 22nd number.
 b. $\frac{456}{2} = 228$ so the median is the 228th number.
 c. $\frac{397}{2} = 198.5$ so the median is the average of the 198th and 199th number.
2. a. $412 + 354 + 212 + 50 + 12 = 1040$, so Tom has seen 1040 cars.
 b. The histogram should like like:



- c. The mode is 1.
 The median is on the $\frac{1040}{2} = 520$ th position. So because $412 + 354 = 766$ is already larger, the median must be 2. The mean is $\frac{412 \times 1 + 354 \times 2 + 212 \times 3 + 50 \times 4 + 12 \times 5}{1040} = 1.9$.
- d. Tom was interested in the average number of people per car. So that would be the mean and is 1.9.
3. a. The mode is 1, since that bar is the highest.
 b. The table should look like:

Days per week	0	1	2	3	4	5	6	7
Frequency	45	55	25	35	10	15	5	35

- c. The median is on place $\frac{225}{2} = 112.5$, so between the 112th and 113th number. $45 + 55 = 100$ and $45 + 55 + 25 = 125$, so both the 112th and 113th number is a 2. So the median is a 2. The mean we can calculate: $\frac{45 \times 0 + 55 \times 1 + 25 \times 2 + 35 \times 3 + 10 \times 4 + 15 \times 5 + 5 \times 6 + 35 \times 7}{225} = 2.7$. So the mean is 2.7.
4. a. If the mode is 3, then the number underneath the stain must be the lower than the frequency of 3. So it must be less than 23.

- b. If also the median is 2.5, then the median must be exactly between the 2 and 3. So the frequency of the 0, 1 and 2 must be equal to the frequency of 3, 4 and 5. 3, 4 and 5 appear $23 + 11 + 5 = 39$ times. 0 and 1 appear $5 + 12 = 17$ times. So 2 must appear $39 - 17 = 22$ times. So the number underneath the stain must be 22.

Checklist

-
- The total amount is odd, so the median is the middle number. Divide the total amount by 2 and round this value up to find the middle number: $\frac{403}{2} \approx 202$, so it is at the 202nd position.
- Add the frequencies in the table, starting with the lowest value, until you get to the 202nd position or past it.
-

★ Exercise

The mode is the most frequent number. In this case, this is 1 (because 1 sibling occurs 1042 times).

The total amount of numbers is 3242. This is an even number, so there is no middle number. If we divide 3242 by 2 we get 1621. So the median will be the mean of the 1621th and 1622th number.

We'll check with the frequency table where these numbers are. 1621 is more than 1042, so the 1621th number isn't in the column corresponding to 0 siblings. $1042 + 579 = 1621$, so the 1621th number is in the column corresponding to 1 sibling. The 1622th number is then in the column corresponding to 2 siblings. So the median is the mean of 1 and 2, which is 1.5.

The mean is

$$\frac{0 \times 579 + 1 \times 1042 + 2 \times 812 + 3 \times 484 + 4 \times 484 + 4 \times 325}{3242} = \frac{5418}{3242} \approx 1.7 \text{ siblings.}$$

Diagnostic Test

1. We need to calculate the percentage of 53 out of 96:

$$\frac{53}{96} \times 100\% \approx 55.2\%$$

So 55.2% of the fourteeners in the USA are in Colorado.

2. a.

$$\begin{aligned} \text{new amount} &= \text{old amount} - 0.09 \times \text{old amount} \\ &= 6.4 - 0.09 \times 6.4 \approx 5.8 \end{aligned}$$

The average mathematics grade in 2019 is a 5.8.

- b.

$$\begin{aligned} \text{new amount} &= \text{old amount} + 0.07 \times \text{old amount} \\ &= 5.8 + 0.07 \times 5.8 \approx 6.2 \end{aligned}$$

The average mathematics grade in 2020 is a 6.2.

- c.

$$\begin{aligned} \text{percentage} &= \frac{\text{increase}}{\text{old amount}} \times 100\% \\ &= \frac{6.6 - 6.4}{6.4} \times 100\% \approx 3.1\% \end{aligned}$$

The grade has increased in 2021 compared to 2018 with 3.1%.

- d. Because the grade in 2021 (6.6) is higher than in 2020 (6.2) it is an increase.

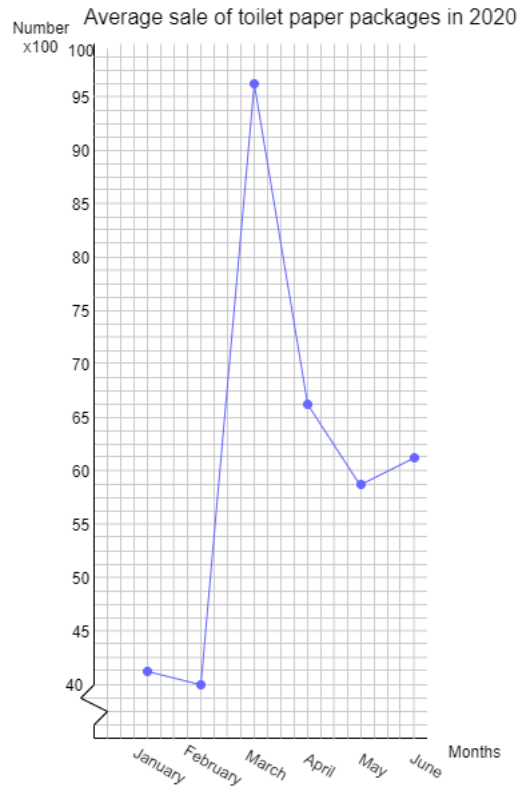
$$\begin{aligned} \text{percentage} &= \frac{\text{increase}}{\text{old amount}} \times 100\% \\ &= \frac{6.6 - 6.2}{6.2} \times 100\% \approx 6.5\% \end{aligned}$$

The grade has increased in 2021 compared to 2020 with a percentage of 6.5%.

3. A possible example could be: Yousef wants to buy new shoes. He goes to the shop and he sees that there is a sale. The shoes he wants are on sale and have a discount of 30%. The original price of the shoes are €45, and he wants to know what the price is he has to pay now.

The idea is that the old amount is 45 and the percentage decrease is 30%. The number of examples you can think of are endless!

4. a. The line chart should look like:



b.

$$\begin{aligned} \text{percentage} &= \frac{\text{increase}}{\text{old amount}} \times 100\% \\ &= \frac{97 - 40}{40} \times 100\% = 142.5\% \end{aligned}$$

The number of toilet paper packages sold increased by 142.5% in March compared to February.

c. We calculate the mean:

$$\frac{4100+4000+9700+6600+5900+6100}{6} \approx 6067$$

On average, 6067 toilet paper packages were sold every month.

5. a. The corresponding table:

Colour	Orange	Blue	Green	Pink	Purple	Yellow	Red	Black
Frequency	6	12	9	7	4	11	8	2

b. Calculate:

$$\text{value} = \frac{6+12}{6+12+9+7+4+11+8+2} \times 100\% \approx 30.5\%$$

30.5% of the children have blue or orange as their favourite colour.

c. Calculate:

$$\text{value} = \frac{9+4+2}{6+12+9+7+4+11+8+2} \times 100\% \approx 25.4\%$$

25.4% of the children have green, purple or black as their favourite colour.

d. You can calculate the corresponding percentage. If the percentage is higher than 50% then Rose is right.

$$\text{Percentage} = \frac{12+11+9}{6+12+9+7+4+11+8+2} \times 100\% \approx 54.2\%$$

Rose is right. More than half the children have blue, yellow or green as their favourite colour. So there are more children who have blue, yellow or green as their favourite colour compared with all the other colours combined.

Another solution:

You can also calculate that 12 children (blue) + 11 children (yellow) + 9 children (green) is a total of 32 children. The number of children who have the other colours as their favourite colour can be calculated by: 6 (orange) + 7 (pink) + 4 (purple) + 8 (red) + 2 (black) = 27 children.

32 children is more than 27 children. So Rose is right.

6. a.
 - i. Calculate the central angles of each sector.
 - ii. Draw a circle and a vertical radius.
 - iii. Draw the central angles clockwise and colour the sections.
 - iv. Write a title and a legend
- b. First calculate the central angles of each sector:

Eindhoven: $\frac{13}{29} \times 360^\circ \approx 161^\circ$

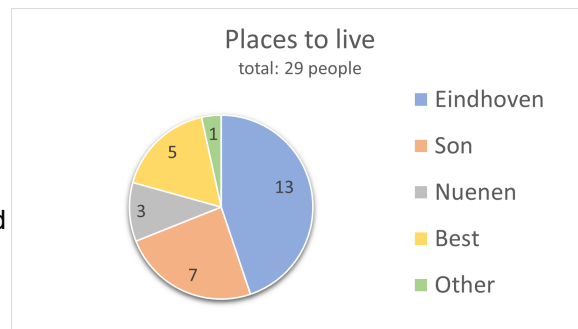
Son: $\frac{7}{29} \times 360^\circ \approx 87^\circ$

Nuenen: $\frac{3}{29} \times 360^\circ \approx 37^\circ$

Best: $\frac{5}{29} \times 360^\circ \approx 62^\circ$

Other: $\frac{1}{29} \times 360^\circ \approx 12^\circ$

Then draw the pie chart. The pie chart should look like this:



7. a. The mode is the most frequent number. So in the diagram you look for the number which occurs the most.
mode for class 2A: 21 (3 times)
mode for class 2B: 14 (4 times)
- b. The median is the middle number of the set of numbers after ordering the numbers from low to high.

Ordering class 2A: 4 4 7 9 13 15 15 21 21 21 22 36 41

median for class 2A: the middle number is 15.

Ordering class 2B: 5 5 8 11 14 14 14 14 23 23 26 32

median for class 2B: there is no middle number, so we have to calculate it by the mean of the two middle numbers. There are in total 12 numbers, so the median is the mean of the 6th and 7th number.

$$\frac{14+14}{2} = 14$$

- c. mean for class 2A:

$$\frac{4+4+7+9+13+15+15+21+21+21+22+36+41}{13} \approx 17.6$$

- mean for class 2B:

$$\frac{5+5+8+11+14+14+14+14+23+23+26+32}{12} \approx 15.8$$

- d. *